THE THEORY

OF

POLITICAL ECONOMY

BY

W. STANLEY JEVONS

LL.D. (EDIN.), M.A. (LOND.), F.R.S.

WITH PREFACE AND NOTES AND AN EXTENSION OF THE BIBLIOGRAPHY OF MATHEMATICAL ECONOMIC WRITINGS

BY

H. STANLEY JEVONS, M.A., B.Sc., F.S.S.

SOMETIME PROFESSOR OF ECONOMICS IN THE UNIVERSITY COLLEGE, CARDIFF, THE UNIVERSITY OF ALLAHABAD AND THE UNIVERSITY OF RANGOON

FIFTH EDITION

First Edition, 1871.
Second Edition, 1879.
Third Edition, 1888.
Fourth Edition, 1911.

PREFACE TO THE FIFTH EDITION

In writing the Preface to the fourth edition of the *Theory of Political Economy* I ventured to predict that it would be the last. That edition was in fact exhausted a few years ago; but Messrs. Macmillan were unable to see their way to reprint the book. I was glad therefore to have an offer from the representative of an American firm to bring out a new edition, for there is a continuing demand for the *Theory*, which has found a place amongst the economic classics of the nineteenth century.

Its appeal lies not merely in the Author's clarity and freshness of diction, but in its combination of two previously separate schools of thought — those favoring respectively the psychological and the mathematical methods in economics. It is still a matter of controversy whether economics should be regarded as a calculus of pleasure and pain or, more correctly, of positive and negative feeling, or whether it should be treated as the science of preferences in the satisfaction of human wants. For myself, I believe that those who refuse or neglect to study the psychological basis of economics as one branch of the science of human behavior are less likely than those who do so to arrive at sound conclusions

tending in their applications to maximise human happiness.

In the preface to the fourth edition I explained that it had been my intention to continue my father's bibliography of mathematico-economic writings by drawing up a bibliography of all books and articles of that kind published from 1880 to 1910; but that as the work was still very far from complete I had decided that the new edition should be issued without it. It was my intention that it should be published later as an independent volume.

Unfortunately, no sooner was the fourth edition published than I became deeply interested in housing reform in the mining valleys of South Wales, and in a few months had resigned from the chair of Economics and Political Science at Cardiff to devote my whole time to housing reform. In 1914, soon after the outbreak of war, I was appointed to the newly established research chair in economics in the University of Allahabad. From 1911 on, therefore, only spasmodic progress was made with the bibliography; and finally I gave up all hope of completing it, though I still have most of the cards.

The purpose of the intended bibliography has been partially served by the publication of *A Select Bibliography of Modern Economic Theory*, 1870 to 1929, edited by H. E. Batson, in the series of bibliographies issued by the London School of Economics. Mention may also be made of the bibliography of mathematico-economic writings prepared by Professor Irving Fisher and published first as an appendix to his article entitled *Cournot and Mathematical Economics* (*Quarterly Journal of Economics, January*, 1898), and later in his edi-

PREFACE TO THE FIFTH EDITION

tion of Cournot's *Researches into the Mathematical Principles of the Theory of Wealth* (Economic Classics Series, 1927). Professor Fisher's bibliography is classified and annotated; and is probably an almost complete list of all published writings on economics of a mathematical character up to 1897. He did not add entries for the later years when reprinting the bibliography in 1927, for, as he wrote in the preface, the mathematical method was now so widely recognised as applicable to economics that the number of publications to be included in the bibliography would have been greatly increased and there was also less need to emphasise its importance.

In preparing the third edition of the *Theory*, my mother, with the assistance of Mr. Foxwell and others, continued the bibliography to 1888 and added a few titles which my father had not discovered. These latter are shown in square brackets. When editing the fourth edition I omitted all titles dated after 1879 as it was my intention to include them in the separate bibliography mentioned above. As this omission may seem to have detracted somewhat from the value of the fourth edition, the part of the bibliography omitted from it is now restored, so that it appears exactly as in the third edition, except for the correction of a few printer's errors.

I close this preface with the hope that looking back to the development of the marginal theory in the nineteenth century will prove as stimulating to students of economics in the United States as in Britain, now that the *Theory of Political Economy* is once more available. I shall not venture to prophesy again as to whether this will be the last edition.

PREFACE TO THE FIRST EDITION

(1871)

THE contents of the following pages can hardly meet with ready acceptance among those who regard the Science of Political Economy as having already acquired a nearly perfect form. I believe it is generally supposed that Adam Smith laid the foundations of this science; that Malthus, Anderson, and Senior added important doctrines; that Ricardo systematised the whole; and, finally, that Mr. J. S. Mill filled in the details and completely expounded this branch of knowledge. Mr. Mill appears to have had a similar notion; for he distinctly asserts that there was nothing in the Laws of Value which remained for himself or any future writer to clear up. Doubtless it is difficult to help feeling that opinions adopted and confirmed by such eminent men have much weight of probability in their favour. Yet, in the other sciences this weight of authority has not been allowed to restrict the free examination of new opinions and theories; and it has often been

ultimately proved that authority was on the wrong side.

There are many portions of Economical doctrine which appear to me as scientific in form as they are consonant with facts. I would especially mention the Theories of Population and Rent, the latter a theory of a distinctly mathematical character, which seems to give a clue to the correct mode of treating the whole science. Had Mr. Mill contented himself with asserting the unquestionable truth of the Laws of Supply and Demand, I should have agreed with him. As founded upon facts, those laws cannot be shaken by any theory; but it does not therefore follow that our conception of Value is perfect and final. Other generally accepted doctrines have always appeared to me purely delusive, especially the so-called Wage Fund Theory. This theory pretends to give a solution of the main problem of the science—to determine the wages of labour; yet, on close examination, its conclusion is found to be a mere truism, namely, that the average rate of wages is found by dividing the whole amount appropriated to the payment of wages by the number of those between whom it is divided. Some other supposed conclusions of the science are of a less harmless character, as, for instance, those regarding the advantage of exchange (see the section on "The Gain by Exchange," p. 142).

In this work I have attempted to treat Economy as a Calculus of Pleasure and Pain, and have

sketched out, almost irrespective of previous opinions, the form which the science, as it seems to me, must ultimately take. I have long thought that as it deals throughout with quantities, it must be a mathematical science in matter if not in language. I have endeavoured to arrive at accurate quantitative notions concerning Utility, Value, Labour, Capital, etc., and I have often been surprised to find how clearly some of the most difficult notions, especially that most puzzling of notions *Value*, admit of mathematical analysis and expression. The Theory of Economy thus treated presents a close analogy to the science of Statical Mechanics, and the Laws of Exchange are found to resemble the Laws of Equilibrium of a lever as determined by the principle of virtual velocities. The nature of Wealth and Value is explained by the consideration of indefinitely small amounts of pleasure and pain, just as the Theory of Statics is made to rest upon the equality of indefinitely small amounts of energy. But I believe that dynamical branches of the Science of Economy may remain to be developed, on the consideration of which I have not at all entered.

Mathematical readers may perhaps think that I have explained some elementary notions, that of the Degree of Utility for instance, with unnecessary prolixity. But it is to the neglect of Economists to obtain clear and accurate notions of quantity and degree of utility that I venture to attribute the present difficulties and imperfections of the science;

and I have purposely dwelt upon the point at full length. Other readers will perhaps think that the occasional introduction of mathematical symbols obscures instead of illustrating the subject. But I must request all readers to remember that, as Mathematicians and Political Economists have hitherto been two nearly distinct classes of persons, there is no slight difficulty in preparing a mathematical work on Economy with which both classes of readers may not have some grounds of complaint.

It is very likely that I have fallen into errors of more or less importance, which I shall be glad to have pointed out; and I may say that the cardinal difficulty of the whole theory is alluded to in the section of Chapter IV. upon the "Ratio of Exchange," beginning at p. 91 (that on "the Law of Indifference," p. 90 of this edition). So able a mathematician as my friend Professor Barker, of Owens College, has had the kindness to examine some of the proof sheets carefully; but he is not, therefore, to be held responsible for the correctness of any part of the work.

My enumeration of the previous attempts to apply mathematical language to Political Economy does not pretend to completeness even as regards English writers; and I find that I forgot to mention a remarkable pamphlet "On Currency" published anonymously in 1840 (London, Charles Knight and Co.) in which a mathematical analysis of the operations of the Money Market is attempted. The

method of treatment is not unlike that adopted by Dr. Whewell, to whose Memoirs a reference is made; but finite or occasionally infinitesimal differences are introduced. On the success of this anonymous theory I have not formed an opinion; but the subject is one which must some day be solved by mathematical analysis. Garnier, in his treatise on Political Economy, mentions several continental mathematicians who have written on the subject of Political Economy; but I have not been able to discover even the titles of their Memoirs.

PREFACE TO THE SECOND EDITION

(1879)

In preparing this second edition certain new sections have been added, the most important of which are those treating of the *dimensions of economic quantities* (pp. 61-69, 83-84, 178-179, 233-235). The subject, of course, is one which lies at the basis of all clear thought about economic science. It cannot be surprising that many debates end in logomachy, when it is still uncertain how many meanings the word *value* has, or what kind of a quantity *utility* itself is. Imagine the mental state of astronomers if they could not agree among themselves whether *Right Ascension* was the name of a heavenly body, or a force or an angular magnitude. Yet this would not be worse than failing to ascertain clearly whether by value we mean a numerical ratio, or a mental state, or a mass of commodity. John Stuart Mill tells us explicitly[1] that "The value of a thing means the quantity of some other thing, or

[1] *Principles of Political Economy*, book iii. chap. vi. sec. i. 1. This definition occurs at the beginning of a carefully prepared summary of the principles of the theory of value.

of things in general, which it exchanges for." It might of course be explained that Mill did not intend what he said; but as the statement stands it makes value into a thing, and is just as philosophic as if one were to say, "Right Ascension means the planet Mars, or planets in general."

These sections upon the dimensions of economic quantities have caused me great perplexity, especially as regards the relation between utility and time (pp. 64-69). The theory of capital and interest also involves some subtleties. I hope that my solutions of the questions raised will be found generally correct; but where they do not settle a question, they may sometimes suggest one which other writers may answer. A correspondent, Captain Charles Christie, R.E., to whom I have shown these sections after they were printed, objects reasonably enough that commodity should not have been represented by M, or Mass, but by some symbol, for instance Q, which would include quantity of space or time or force, in fact almost any kind of quantity. Services often involve time, or force exerted, or space passed over, as well as mass. In this objection I quite concur, and I must therefore request the reader either to interpret M with a wider meaning than is given to it in p. 64, or else mentally to substitute another symbol.

In treating the dimensions of interest, I point out the curious fact that so profound a mathematician as the late Dean Peacock went quite astray upon the subject (pp. 250-253). Other new sections are those

in which I introduce the idea of negative and approximately zero value, showing that negative value may be brought under the forms of the equations of exchange without any important modification. Readers of Mr. Macleod's works are of course familiar with the idea of negative value; but it was desirable for me to show how important it really is, and how naturally it falls in with the principles of the theory. I may also draw attention to the section (pp. 102-106) in which I illustrate the mathematical character of the equations of exchange by drawing an exact analogy between them and the equations applying to the equilibrium of the lever.

Two or three correspondents, especially Herr Harald Westergaard of Copenhagen, have pointed out that a little manipulation of the symbols, in accordance with the simple rules of the differential calculus, would often give results which I have laboriously argued out. The whole question is one of maxima and minima, the mathematical conditions of which are familiar to mathematicians. But, even if I were capable of presenting the subject in the concise symbolic style satisfactory to the taste of a practised mathematician, I should prefer in an essay of this kind to attain my results by a course of argument which is not only fundamentally true, but is clear and convincing to many readers who, like myself, are not skilful and professional mathematicians. In short, I do not write for mathematicians, nor as a mathematician, but as an economist wishing to convince

other economists that their science can only be satisfactorily treated on an explicitly mathematical basis. When mathematicians recognise the subject as one with which they may usefully deal, I shall gladly resign it into their hands. I have expressed a feeling in more than one place that the whole theory might probably have been put in a more general form by treating labour as a negative utility, and thus bringing it under the ordinary equations of exchange. But the fact is there is endless occupation for an economist in developing and improving his science, and I have found it requisite to reissue this essay, as the bibliopoles say, "with all faults." I have, however, carefully revised every page of the book, and have reason to hope that little or no real error remains in the doctrines stated. The faults are in the form rather than the matter.

Among minor alterations, I may mention the substitution for the name Political Economy of the single convenient term *Economics*. I cannot help thinking that it would be well to discard, as quickly as possible, the old troublesome double-worded name of our Science. Several authors have tried to introduce totally new names, such as Plutology, Chrematistics, Catallactics, etc. But why do we need anything better than Economics? This term, besides being more familiar and closely related to the old term, is perfectly analogous in form to *Mathematics*, *Ethics*, *Æsthetics*, and the names of various other branches of knowledge, and it has moreover the

authority of usage from the time of Aristotle. Mr. Macleod is, so far as I know, the re-introducer of the name in recent years, but it appears to have been adopted also by Mr. Alfred Marshall at Cambridge. It is thus to be hoped that *Economics* will become the recognised name of a science, which nearly a century ago was known to the French Economists as *la science économique*. Though employing the new name in the text, it was obviously undesirable to alter the title-page of the book.

When publishing a new edition of this work, eight years after its first appearance, it seems natural that I should make some remarks upon the changes of opinion about economic science which have taken place in the interval. A remarkable discussion has been lately going on in the reviews and journals concerning the logical method of the science, touching even the question whether there exists any such science at all. Attention was drawn to the matter by Mr. T. E. Cliffe Leslie's remarkable article[1] "On the Philosophical Method of Political Economy," in which he endeavours to dissipate altogether the deductive science of Ricardo. Mr. W. T. Thornton's writings have a somewhat similar tendency. The question has been further stirred up by the admirable criticism to which it was subjected in the masterly address of Professor J. K. Ingram, at the last meeting

[1] *Hermathena*, No. iv., 1876, pp. 1-32. Republished in Mr. Leslie's collected *Essays in Political and Moral Philosophy*, Dublin, 1879, pp. 216-242.

of the British Association. This Address has been reprinted in several publications[1] in England, and has been translated into the chief languages of Western Europe. It is evident, then, that a spirit of very active criticism is spreading, which can hardly fail to overcome in the end the prestige of the false old doctrines. But what is to be put in place of them? At the best it must be allowed that the fall of the old orthodox creed will leave a chaos of diverse opinions. Many would be glad if the supposed science collapsed altogether, and became a matter of history, like astrology, alchemy, and the occult sciences generally. Mr. Cliffe Leslie would not go quite so far as this, but would reconstruct the science in a purely inductive or empirical manner. Either it would then be a congeries of miscellaneous disconnected facts, or else it must fall in as one branch of Mr. Spencer's Sociology. In any case, I hold that *there must arise a science of the development of economic forms and relations.*

But as regards the fate of the deductive method, I disagree altogether with my friend Mr. Leslie; he is in favour of simple deletion; I am for thorough reform and reconstruction. As I have previously explained,[2] the present chaotic state of Economics

[1] *Journal of the London Statistical Society*, December 1878, vol. xli. pp. 602-629. *Journal of the Statistical and Social Inquiry Society of Ireland*, August 1878, vol. vii. Appendix. Also as a separate publication, Longmans, London, 1878.

[2] "The Future of Political Economy," *Fortnightly Review*, November 1876, vol. viii., N.S., pp. 617-631. Translated in the *Journal des Économistes*, March 1877, 3me Série, vol. xlv., p. 325.

arises from the confusing together of several branches of knowledge. Subdivision is the remedy. We must distinguish the empirical element from the abstract theory, from the applied theory, and from the more detailed art of finance and administration. Thus will arise various sciences, such as commercial statistics, the mathematical theory of economics, systematic and descriptive economics, economic sociology, and fiscal science. There may even be a kind of cross subdivision of the sciences; that is to say, there will be division into branches as regards the subject, and division according to the manner of treating the branch of the subject. The manner may be theoretical, empirical, historical, or practical; the subject may be capital and labour, currency, banking, taxation, land tenure, etc.—not to speak of the more fundamental division of the science as it treats of consumption, production, exchange, and distribution of wealth. In fact, the whole subject is so extensive, intricate, and diverse, that it is absurd to suppose it can be treated in any single book or in any single manner. It is no more one science than statics, dynamics, the theory of heat, optics, magneto-electricity, telegraphy, navigation, and photographic chemistry are one science. But as all the physical sciences have their basis more or less obviously in the general principles of mechanics, so all branches and divisions of economic science must be pervaded by certain general principles. It is to the investigation of such principles—to the tracing out of the mechanics

of self-interest and utility, that this essay has been devoted. The establishment of such a theory is a necessary preliminary to any definite drafting of the superstructure of the aggregate science.

Turning now to the theory itself, the question is not so much whether the theory given in this volume is true, but whether there is really any novelty in it. The exclusive importance attributed in England to the Ricardian School of Economists, has prevented almost all English readers from learning the existence of a series of French, as well as a few English, German, or Italian economists, who had from time to time treated the science in a more or less strictly mathematical manner. In the first edition (pp. 14-18), I gave a brief account of such writings of the kind as I was then acquainted with; it is from the works there mentioned, if from any, that I derived the idea of investigating Economics mathematically. To Lardner's *Railway Economy* I was probably most indebted, having been well acquainted with that work since the year 1857. Lardner's book has always struck me as containing a very able investigation, the scientific value of which has not been sufficiently estimated; and in chapter xiii. (pp. 286-296, etc.) we find the Laws of Supply and Demand treated mathematically and illustrated graphically.

In the preface to the first edition (p. xi),[1] I remarked that in his treatise on Political Economy, M. Joseph Garnier mentioned several continental

[1] See p. ix of this edition.

mathematicians who had written on the subject of Economics, and I added that I had not been able to discover even the titles of their memoirs. This, however, must have been the result of careless reading or faulty memory, for it will be found that Garnier himself[1] mentions the titles of several books and memoirs. The fact is that, writing as I did then at a distance from any large library, I made no attempt to acquaint myself with the literature of the subject, little thinking that it was so copious and in some cases so excellent as is now found to be the case. With the progress of years, however, my knowledge of the literature of political economy has been much widened, and the hints of friends and correspondents have made me aware of the existence of many remarkable works which more or less anticipate the views stated in this book. While preparing this new edition, it occurred to me to attempt the discovery of all existing writings of the kind. With this view I drew up a chronological list of all the mathematico-economic works known to me, already about seventy in number, which list, by the kindness of the editor, Mr. Giffen, was printed in the *Journal of the London Statistical Society* for June 1878 (vol. xli. pp. 398-401), separate copies being forwarded to the leading economists, with a request for additions and corrections. My friend, M. Léon Walras, Rector of the Academy of Lausanne, after himself making considerable additions to the list, communicated it to

[1] *Traité d'Économie Politique*, 5me éd., Paris, 1863, pp. 700-2.

the *Journal des Économistes* (December 1878), to the editor of which we are much indebted for its publication. Copies of the list were also sent to German and Italian economical journals. For the completion of the bibliographical list I am under obligations to Professor W. B. Hodgson, Professor Adamson, Mr. W. H. Brewer, M.A., H.M. Inspector of Schools, the Baron d'Aulnis de Bourouill, Professor of Political Economy at Utrecht, M. N. G. Pierson of Amsterdam, M. Vissering of Leiden, Professor Luigi Cossa of Pavia, and others.

All reasonable exertions have thus been made to render complete and exhaustive the list of mathematico-economic works and papers, which is now printed in the first [now fifth] Appendix to this book (pp. 322-339). It is hardly likely that many additions can be made to the earlier parts of the lists, but I shall be much obliged to any readers who can suggest corrections or additions. I shall also be glad to be informed of any new publications suitable for insertion in the list. On the other hand, it is possible that some of the books mentioned in the list ought not to be there. I have not been able in all cases to examine the publications myself, so that some works inserted at the suggestion of correspondents may have been named under misconception of the precise purpose of the list. Economic works, for instance, containing numerical illustrations and statistical facts numerically expressed, however abundantly, have not been intentionally included, unless there was also mathematical method in the reasoning. Without this condition the whole literature of numerical commercial statistics would have been imported into my list. In other cases only a small portion of a book named can be called mathematico-economic; but this fact is generally noted by the quotation of the chapters or pages in question. The tendency, however, has been to include rather than to exclude, so that the reader might have before him the whole field of literature requiring investigation.

To avoid misapprehension it may be well to explain that the ground for inserting any publication or part of a publication in this list, is its containing *an explicit recognition of the mathematical character of economics, or the advantage to be attained by its symbolical treatment.* I contend that all economic writers must be mathematical so far as they are scientific at all, because they treat of economic quantities, and the relations of such quantities, and all quantities and relations of quantities come within the scope of the mathematics. Even those who have most strongly and clearly protested against the recognition of their own method, continually betray in their language the quantitative character of their reasonings. What, for instance, can be more clearly mathematical in matter than the following quotation from Cairnes's chief work:[1]—"We can have no difficulty in seeing how cost in its principal

[1] *Some Leading Principles of Political Economy Newly Expounded*, pt. 1, chap. iii. p. 97.

elements is to be computed. In the case of labour, the cost of producing a given commodity will be represented by the number of average labourers employed in its production—regard at the same time being had to the severity of the work and the degree of risk it involves—multiplied by the duration of their labours. In that of abstinence, the principle is analogous: the sacrifice will be measured by the quantity of wealth abstained from, taken in connection with the risk incurred, and multiplied by the duration of the abstinence." Here we deal with computation, multiplication, degree of severity, degree of risk, quantity of wealth, duration, etc., all essentially mathematical things, ideas, or operations. Although my esteemed friend and predecessor has in his preliminary chapter expressly abjured my doctrines, he has unconsciously adopted the mathematical method in all but appearance.

We might easily go further back, and discover that even the father of the science, as he is often considered, is thoroughly mathematical. In the fifth chapter of the First Book of the *Wealth of Nations*, for instance, we find Adam Smith continually arguing about "quantities of labour," "measures of value," "measures of hardship," "proportion," "equality," etc.; the whole of the ideas in fact are mathematical. The same might be said of almost any other passages from the scientific parts of the treatise, as distinguished from the historical parts. In the first chapter of the Second Book (29th paragraph), we read—"The produce of land, mines, and fisheries, when their natural fertility is equal, is in proportion to the extent and proper application of the capitals employed about them. When the capitals are equal, and equally well applied, it is in proportion to their natural fertility." Now every use of the word *equal* or *equality* implies the existence of a mathematical equation; an equation is simply an equality; and every use of the word proportion implies a ratio expressible in the form of an equation.

I hold, then, that to argue mathematically, whether correctly or incorrectly, constitutes no real differentia as regards writers on the theory of economics. But it is one thing to argue and another thing to understand and to recognise explicitly the method of the argument. As there are so many who talk prose without knowing it, or, again, who syllogise without having the least idea what a syllogism is, so economists have long been mathematicians without being aware of the fact. The unfortunate result is that they have generally been bad mathematicians, and their works must fall. Hence the explicit recognition of the mathematical character of the science was an almost necessary condition of any real improvement of the theory. It does not follow, of course, that to be explicitly mathematical is to ensure the attainment of truth, and in such writings as those of Canard and Whewell, we find plenty of symbols and equations with no result of value, owing to the fact that they simply translated into symbols the doctrines obtained, and

erroneously obtained, without their use. Such writers misunderstood and inverted altogether the function of mathematical symbols, which is to guide our thoughts in the slippery and complicated processes of reasoning. Ordinary language can usually express the first axioms of a science, and often also the matured results; but only in the most lame, obscure, and tedious way can it lead us through the mazes of inference.

The bibliographical list, of which I am speaking, is no doubt a very heterogeneous one, and may readily be decomposed into several distinct classes of economic works. In a first class may be placed the writings of those economists who have not at all attempted mathematical treatment in an express or systematic manner, but who have only incidentally acknowledged its value by introducing symbolic or graphical statements. Among such writers may be mentioned especially Rau (1868), Hagen (1844), J. S. Mill (1848), and Courcelle-Seneuil (1867). Many readers may be surprised to hear that John Stuart Mill has used mathematical symbols; but, on turning to Book III., chapters xvii. and xviii., of the *Principles of Political Economy*, those difficult and tedious chapters in which Mill leads the reader through the Theory of International Trade and International Values, by means of yards of linen and cloth, the reader will find that Mill at last yields, and expresses himself concisely and clearly[1] by means of equations between m, n, p, and q. His mathematics

[1] Book iii. chap. xviii. sec. 7.

are very crude; still there is some approach to a correct mathematical treatment, and the result is that these chapters, however tedious and difficult, will probably be found the truest and most enduring parts of the whole treatise.

A second class of economists contains those who have abundantly employed mathematical apparatus, but, misunderstanding its true use, or being otherwise diverted from a true theory, have built upon the sand. Misfortunes of this kind are not confined to the science of economics, and in the most exact branches of physical science, such as mechanics, molecular physics, astronomy, etc., it would be possible to adduce almost innumerable mathematical treatises, which must be pronounced nonsense. In the same category must be placed the mathematical writings of such economists as Canard (1801), Whewell (1829, 1831, and 1850), Esmenard du Mazet (1849 and 1851), and perhaps Du Mesnil-Marigny (1860).

The third class forms an antithesis with the second, for it contains those authors who, without any parade of mathematical language or method, have nevertheless carefully attempted to reach precision in their treatment of quantitative ideas, and have thus been led to a more or less complete comprehension of the true theory of utility and wealth. Among such writers Francis Hutcheson, the Irish founder of the great Scotch School, and the predecessor of Adam Smith at Glasgow, probably stands first. His

employment of mathematical symbols[1] seems rather crude and premature, but the precision of his ideas about the estimation of quantities of good and evil is beyond praise. He thoroughly anticipates the foundations of Bentham's moral system, showing that the Moment of Good or Evil is, in a compound proportion of the Duration and Intenseness, affected also by the Hazard or uncertainty of our existence.[2] As to Bentham's ideas, they are adopted as the starting-point of the theory given in this work, and are quoted at the beginning of chapter ii. (pp. 28-9). Bentham has repeated his statement as to the mode of measuring happiness in several different works and pamphlets, as for instance in that remarkable one called "A Table of the Springs of Action" (London, 1817, p. 3); and also in the "Codification Proposal, addressed by Jeremy Bentham to all Nations professing Liberal Opinions" (London, 1822, pp. 7-11). He here speaks explicitly of the application of *arithmetic* to questions of utility, meaning no doubt the application of mathematical methods. He even describes (p. 11) the four circumstances governing the value of a pleasure or pain as the *dimensions* of its value, though he is incorrect in treating *propinquity* and *certainty* as dimensions.

It is worthy of notice that Destutt de Tracy, one of the most philosophic of all economists, has in a few words recognised the true method of treatment,

[1] 1720. Hutcheson. *An Inquiry*, 1729, etc., pp. 186-198.

[2] 1728. Hutcheson. *An Essay*, etc., pp. 34-43, and elsewhere.

though he has not followed up his own idea. Referring to the circumstances which, in his opinion, render all economic and moral calculations very delicate, he says,[1] "On ne peut guère employer dans ces matières que des considérations tirées de la théorie des limites." So well known an English economist as Malthus has also shown in a few lines his complete appreciation of the mathematical nature of economic questions. In one of his excellent pamphlets[2] he remarks, "Many of the questions, both in morals and politics, seem to be of the nature of the problems *de maximis et minimis* in Fluxions; in which there is always a point where a certain effect is the greatest, while on either side of this point it gradually diminishes." But I have not thought it desirable to swell the bibliographical list by including all the works in which there are to be found brief or casual remarks of the kind.

I may here remark that all the writings of Mr. Henry Dunning Macleod exhibit a strong tendency to mathematical treatment. Some of his works or papers in which this mathematical spirit is most strongly manifested have been placed in the list. It is not my business to criticise his ingenious views, or

[1] *Élémens d'Ideologie*, iv^e^ et v^e^ Parties. *Traité de la Volonté et de ses effets*, Paris, 1815, 8vo, p. 499. Edition of 1826, p. 335. American Edition, *A Treatise on Political Economy, translated from the unpublished French original.* Georgetown, D.C., 1817, p. xiii.

[2] *Observations on the Effects of the Corn Laws, and of a rise or fall in the price of Corn on the Agriculture and General Wealth of the Country.* London, 1814, p. 30; 3rd ed., 1815, p. 32.

to determine how far he really has created a mathematical system. While I certainly differ from him on many important points, I am bound to acknowledge the assistance which I derive from the use of several of his works.

In the fourth and most important class of mathematico-economic writers must be placed those who have consciously and avowedly attempted to frame a mathematical theory of the subject, and have, if my judgment is correct, succeeded in reaching a true view of the Science. In this class certain distinguished French philosophers take precedence and priority. One might perhaps go back with propriety to Condillac's work, *Le Commerce et le Gouvernement*, first published in the year 1776, the same year in which the *Wealth of Nations* appeared. In the first few chapters of this charming philosophic work we meet perhaps the earliest distinct statement of the true connection between value and utility. The book, however, is not included in the list because there is no explicit attempt at mathematical treatment. It is the French engineer Dupuit who must probably be credited with the earliest perfect comprehension of the theory of utility. In attempting to frame a precise measure of the utility of public works, he observed that the utility of a commodity not only varies immensely from one individual to another, but that it is also widely different for the same person according to circumstances. He says, "nous verrions que l'utilité du morceau de pain peut croître pour le

même individu depuis zéro jusqu'au chiffre de sa fortune entière" (1849, Dupuit, *De l'influence des Péages*, etc., p. 185). He establishes, in fact, a theory of the *gradation of utility*, beautifully and perfectly expounded by means of geometrical diagrams, and this theory is undoubtedly coincident in essence with that contained in this book. He does not, however, follow his ideas out in an algebraic form. Dupuit's theory was the subject of some controversy in the pages of the *Annales des Ponts et Chaussées*, but did not receive much attention elsewhere, and I am not aware that any English economist ever knew anything about these remarkable memoirs.

The earlier treatise of Cournot, his admirable *Recherches sur les principes mathématiques de la théorie des richesses* (Paris, 1838), resembles Dupuit's memoirs in being, until within the last few years, quite unknown to English economists. In other respects Cournot's method is contrasted to Dupuit's. Cournot did not frame any ultimate theory of the ground and nature of utility and value, but, taking the palpable facts known concerning the relations of price, production and consumption of commodities, he investigated these relations analytically and diagraphically with a power and felicity which leaves little to be desired. This work must occupy a remarkable position in the history of the subject. It is strange that it should have remained for me among Englishmen to discover its value. Some years since (1875) Mr. Todhunter wrote to me

as follows: "I have sometimes wondered whether there is anything of importance in a book published many years since by M. A. A. Cournot, entitled *Recherches sur les principes mathématiques de la théorie des richesses.* I never saw it, and when I have mentioned the title, I never found any person who had read the book. Yet Cournot was eminent for mathematics and metaphysics, and so there may be some merit in this book." I procured a copy of the work as far back as 1872, but have only recently studied it with sufficient care to form any definite opinion upon its value. Even now I have by no means mastered all parts of it, my mathematical power being insufficient to enable me to follow Cournot in all parts of his analysis. My impression is that the first chapter of the work is not remarkable; that the second chapter contains an important anticipation of discussions concerning the proper method of treating prices, including an anticipation (p. 21) of my logarithmic method of ascertaining variations in the value of gold; that the third chapter, treating of the conditions of the foreign exchanges, is highly ingenious if not particularly useful; but that by far the most important part of the book commences with the fourth chapter upon the "Loi du débit." The remainder of the book, in fact, contains a wonderful analysis of the laws of supply and demand, and of the relations of prices, production, consumption, expenses and profits. Cournot starts from the assumption that the débit or

demand for a commodity is a function of the price, or $D = F(p)$; and then, after laying down empirically a few conditions of this function, he proceeds to work out with surprising power the consequences which follow from those conditions. Even apart from its economic importance, this investigation, so far as I can venture to judge it, presents a beautiful example of mathematical reasoning, in which knowledge is apparently evolved out of ignorance. In reality the method consists in assuming certain simple conditions of the functions as conformable to experience, and then disclosing by symbolic inference the implicit results of these conditions. But I am quite convinced that the investigation is of high economic importance, and that, when the parts of political economy to which the theory relates come to be adequately treated, as they never have yet been, the treatment must be based upon the analysis of Cournot, or at least must follow his general method. It should be added that his investigation has little relation to the contents of this work, because Cournot does not recede to any theory of utility, but commences with the phenomenal laws of supply and demand.

Discouraged apparently by the small amount of attention paid to his mathematical treatise, Cournot in a later year (1863) produced a more popular non-symbolic work on Economics; but this later work does not compare favourably in interest and importance with his first treatise.

English economists can hardly be blamed for their

ignorance of Cournot's economic works when we find French writers equally bad. Thus the authors of Guillaumin's excellent *Dictionnaire de l'Économie Politique*, which is on the whole the best work of reference in the literature of the science, ignore Cournot and his works altogether, and so likewise does Sandelin in his copious *Répertoire Général d'Économie Politique*. M. Joseph Garnier in his otherwise admirable text-book[1] mixes up Cournot with far inferior mathematicians, saying: "Dans ces derniers temps M. Esmenard du Mazet, et M. du Mesnil-Marigny ont aussi fait abus, ce nous semble, des formules algébriques; les Recherches sur les Principes Mathématiques des Richesses de M. Cournot, ne nous ont fourni aucun moyen d'élucidation." MacCulloch of course knows nothing of Cournot. Mr. H. D. Macleod has the merit at least of mentioning Cournot's work, but he misspells the name of the author, and gives only the title of the book, which he had probably never seen.

We now come to a truly remarkable discovery in the history of this branch of literature. Some years since my friend Professor Adamson had noticed in one of Kautz's works on Political Economy[2] a brief reference to a book said to contain a theory of pleasure and pain, written by a German author named Hermann Heinrich Gossen. Although he had advertised for it, Professor Adamson was unable to

[1] *Traité d'Économie Politique*, cinquième édition, p. 701.

[2] *Theorie und Geschichte der National-Oekonomik*, 1858, vol. i. p. 9.

obtain a sight of this book until August 1878, when he fortunately discovered it in a German bookseller's catalogue, and succeeded in purchasing it. The book was published at Brunswick in 1854; it consists of 278 well-filled pages, and is entitled, *Entwickelung der Gesetze des menschlichen Verkehrs, und der daraus fliessenden Regeln für menschliches Handeln*, which may be translated—"Development of the laws of Human Commerce, and of the consequent Rules of Human Action." I will describe the contents of this remarkable volume as they are reported to me by Professor Adamson.

Gossen evidently held the highest possible opinion of the importance of his own theory, for he commences by claiming honours in economic science equal to those of Copernicus in astronomy. He then at once insists that mathematical treatment, being the only sound one, must be applied throughout; but, out of consideration for the reader, the higher analysis will be explicitly introduced only when it is requisite to determine maxima and minima. The treatise then opens with the consideration of Economics as the theory of pleasure and pain, that is as the theory of the procedure by which the individual and the aggregate of individuals constituting society, may realise the maximum of pleasure with the minimum of painful effort. The natural law of pleasure is then clearly stated, somewhat as follows: *Increase of the same kind of consumption yields pleasure continuously diminishing*

up to the point of satiety. This law he illustrates geometrically, and then proceeds to investigate the conditions under which the total pleasure from one or more objects may be raised to a maximum.

The term *Werth* is next introduced, which may, Professor Adamson thinks, be rendered with strict accuracy as *utility*, and Gossen points out that the quantity of utility, material or immaterial, is measured by the quantity of pleasure which it affords. He classifies useful objects as: (1) those which possess pleasure-giving powers in themselves; (2) those which only possess such powers when in combination with other objects; (3) those which only serve as means towards the production of pleasure-giving objects. He is careful to point out that there is no such thing as absolute utility, utility being purely a relation between a thing and a person. He next proceeds to give the derivative laws of utility somewhat in the following manner:—That separate portions of the same pleasure-giving object have very different degrees of utility, and that in general for each person only a limited number of such portions has utility; any addition beyond this limit is useless, but the point of uselessness is only reached after the utility has gone through all the stages or degrees of intensity. Hence he draws the practical conclusion that each person should so distribute his resources as to render the final increments of each pleasure-giving commodity of equal utility for him.

In the next place Gossen deals with labour,

starting from the proposition that the utility of any product must be estimated after deduction of the pains of labour required to produce it. He describes the variation of the pain of labour much as I have done, exhibiting it graphically, and inferring that we must carry on labour to the point at which the utility of the product equals the pain of production. In treating the theory of exchange he shows how barter gives rise to an immense increase of utility, and he infers that exchange will proceed up to the point at which the utilities of the portions next to be given and received are equal. A complicated geometrical representation of the theory of exchange is given. The theory of rent is investigated in a most general manner, and the work concludes with somewhat vague social speculations, which, in Professor Adamson's opinion, are of inferior merit compared with the earlier portions of the treatise.

From this statement it is quite apparent that Gossen has completely anticipated me as regards the general principles and method of the theory of Economics. So far as I can gather, his treatment of the fundamental theory is even more general and thorough than what I was able to scheme out. In discussing the book, I lie under the serious difficulty of not being able to read it; but, judging from what Professor Adamson has written or read to me, and from an examination of the diagrams and symbolic parts of the work, I should infer that Gossen has been unfortunate in the development of his theory.

Instead of dealing, as Cournot and myself have done, with undetermined functions, and introducing the least possible amount of assumption, Gossen assumed, for the sake of simplicity, that economic functions follow a linear law, so that his curves of utility are generally taken as straight lines. This assumption enables him to work out a great quantity of precise formulas and tabular results, which fill many pages of the book. But, inasmuch as the functions of economic science are seldom or never really linear, and usually diverge very far from the straight line, I think that the symbolic and geometric illustrations and developments introduced by Gossen must for the most part be put down among the many products of misplaced ingenuity. I may add, in my own behalf, that he does not seem really to reach the equations of exchange as established in this book; that the theory of capital and interest is wanting; and that there is a total absence of any resemblance between the working out of the matter, except such as arises from a common basis of truth.

The coincidence, however, between the essential ideas of Gossen's system and my own is so striking, that I desire to state distinctly, in the first place, that I never saw nor so much as heard any hint of the existence of Gossen's book before August 1878, and to explain, in the second place, how it was that I did not do so. My unfortunate want of linguistic power has prevented me, in spite of many attempts, from ever becoming familiar enough with German to

read a German book. I once managed to spell out with assistance part of the logical lecture notes of Kant; but that is my sole achievement in German literature. Now this work of Gossen has remained unknown even to most of the great readers of Germany. Professor Adamson remarks that the work seems to have attracted no attention in Germany. The eminent and learned economist of Amsterdam, Professor N. G. Pierson, writes to me: "Gossen's book is totally unknown to me. Roscher does not mention it in his very laborious *History of Political Economy in Germany*. I never saw it quoted; but I will try to get it. It is very curious that such a remarkable work has remained totally unknown even to a man like Professor Roscher, who has read everything." Mr. Cliffe Leslie, also, who has made the German Economists his special study, informs me that he was quite unaware of the existence of the book.[1] Under such circumstances it would have been far more probable that I should discover the theory of pleasure and pain, than that I should discover Gossen's book, and I have carefully pointed out, both in the first edition and in this, certain passages of Bentham, Senior, Jennings, and other authors, from which my system was, more or less consciously, developed. I cannot claim to be

[1] A copy of Gossen's book will be found in the Library of the British Museum (Press mark 8408. cc. 16). It was not acquired by that institution until May 24, 1865, as shown by the date stamped upon the copy.

totally indifferent to the rights of priority; and from the year 1862, when my theory was first published in brief outline, I have often pleased myself with the thought that it was at once a novel and an important theory. From what I have now stated in this preface it is evident that novelty can no longer be attributed to the leading features of the theory. Much is clearly due to Dupuit, and of the rest a great share must be assigned to Gossen. Regret may easily be swallowed up in satisfaction if I succeed eventually in making that understood and valued which has been so sadly neglected.

Almost nothing is known to me concerning Gossen; it is uncertain whether he is living or not. On the title-page he describes himself as "königlich preussischem Regierungs-Assessor ausser Dienst," which may be translated "Royal Prussian Government Assessor, retired"; but the tone of his remarks here and there seems to indicate that he was a disappointed if not an injured man. The reception of his one work can have lent no relief to these feelings; rather it must much have deepened them. The book seems to have contained his one cherished theory; for I can find under the name of Gossen no trace of any other publication or scientific memoir whatever. The history of these forgotten works is, indeed, a strange and discouraging one; but the day must come when the eyes of those who cannot see will be opened. Then will due honour be given to all who like Cournot and Gossen have laboured in a

thankless field of human knowledge, and have met with the neglect or ridicule they might well have expected. Not indeed that such men do really work for the sake of honour; they bring forth a theory as the tree brings forth its fruit.

It remains for me to refer to the mathematico-economic writings of M. Léon Walras, the Rector of the Academy of Lausanne. It is curious that Lausanne, already distinguished by the early work of Isnard (1781), should recently have furnished such important additions to the science as the Memoirs of Walras. For important they are, not only because they complete and prove that which was before published elsewhere in the works described above, but because they contain a third or fourth independent discovery of the principles of the theory. If we are to trace out "the filiation of ideas" by which M. Walras was led to his theory, we should naturally look back to the work of his father, Auguste Walras, published at Paris in 1831, and entitled *De la nature de la richesse, et de l'origine de la valeur*. In this work we find, it is true, no distinct recognition of the mathematical method, but the analysis of value is often acute and philosophic. The principal point of the work moreover is true, that value depends upon *rarity*—"La valeur," says Auguste Walras, "dérive de la rareté." Now it is precisely upon this idea of the degree of rarity of commodities that Léon Walras bases his system. The fact that some four or more

independent writers such as Dupuit, Gossen, Walras, and myself should in such different ways have reached substantially the same views of the fundamental ideas of economic science, cannot but lend great probability, not to say approximate certainty, to those views. I am glad to hear that M. Walras intends to bring out a new edition of his Mathematico-Economic Memoirs, to which the attention of my readers is invited. The titles of his publications will be found in the Appendix I. [V. of this edition].

The works of Von Thünen and of several other German economists contain mathematical investigations of much interest and importance. A considerable number of such works will be found noted in the list, which, however, is especially defective as regards German literature. I regret that I am not able to treat this branch of the subject in an adequate manner.

My bibliographical list shows that in recent years, that is to say since the year 1873, there has been a great increase in the number of mathematico-economic writings. The names of Fontaneau, Walras, Avigdor, Lefèvre, Petersen, Boccardo, recur time after time. In such periodicals as the *Journal des Actuaires français*, or the *National-Oekonomisk Tidsskrift*—a journal so creditable to the energy and talent of the Danish Economic School—the mathematical theory of Economics is treated as one of established interest and truth, with which readers would naturally be acquainted. In England we have

absolutely no periodical in which such discussions could be conducted. The reader will not fail to remark that it is into the hands of French, Italian, Danish, or Dutch writers that this most important subject is rapidly passing. They will develop that science which only excites ridicule and incredulity among the followers of Mill and Ricardo. There are just a few English mathematicians, such as Fleeming Jenkin, George Darwin, Alfred Marshall, or H. D. Macleod, and one or two Americans like Professor Simon Newcomb, who venture to write upon the obnoxious subject of mathematico-economic science. I ought to add, however, that at Cambridge (England) the mathematical treatment of Economics is becoming gradually recognised owing to the former influence of Mr. Alfred Marshall, now the Principal of University College, Bristol, whose ingenious mathematico-economic problems, expounded *more geometrico*, have just been privately printed at Cambridge.

If we overlook Hutcheson, who did not expressly write on Economics, the earliest mathematico-economic author seems to be the Italian Ceva, whose works have just been brought to notice in the *Giornale degli Economisti* (see 1878, Nicolini). Ceva wrote in the early part of the eighteenth century, but I have as yet no further information about him. The next author in the list is the celebrated Beccaria, who printed a very small, but distinctly mathematical, tract on Taxation as early as 1765. Italians were

thus first in the field. The earliest English work of the kind yet discovered is an anonymous *Essay on the Theory of Money,* published in London in 1771, five years before the era of the *Wealth of Nations.* Though crude and absurd in some parts, it is not devoid of interest and ability, and contains a distinct and partially valid attempt to establish a mathematical theory of currency. This remarkable Essay is, so far as I know, wholly forgotten and almost lost in England. Neither MacCulloch nor any other English economist known to me, mentions the work. I discovered its existence a few months ago by accidentally finding a copy on a bookseller's stall. But it shames an Englishman to learn that English works thus unknown in their own country are known abroad, and I owe to Professor Luigi Cossa, of the University of Pavia, the information that the Essay was written by Major-General Henry Lloyd, an author of some merit in other branches of literature. Signor Cossa's excellent *Guido alla studio di Economia Politica,* a concise but judiciously written text-book, is well qualified to open our eyes as to the insular narrowness of our economic learning. It is a book of a kind much needed by our students of Economics, and I wish that it could be published in an English dress.

From this bibliographical survey emerges the wholly unexpected result, that the mathematical treatment of Economics is coeval with the science itself. The notion that there is any novelty or

originality in the application of mathematical methods or symbols must be dismissed altogether. While there have been political economists there has always been a certain number who with various success have struck into the unpopular but right path. The unfortunate and discouraging aspect of the matter is the complete oblivion into which this part of the literature of Economics has always fallen, oblivion so complete that each mathematico-economic writer has been obliged to begin almost *de novo.* It is with the purpose of preventing for the future as far as I can such ignorance of previous exertions, that I have spent so much pains upon this list of books.

I should add that in arranging the list I have followed, very imperfectly, the excellent example set by Professor Mansfield Merriman, of the Sheffield Scientific School of Yale College, in his "List of Writings relating to the Method of Least Squares."[1] Such bibliographies are of immense utility, and I hope that the time is nearly come when each student of a special branch of science or literature will feel bound to work out its bibliography, unless, of course, the task shall have been already accomplished. The reader will see that, in Appendix II. [IV. of this edition], I have taken the liberty of working out also a part of the bibliography of my own writings.

Looking now to the eventual results of the theory, I must beg the reader to bear in mind that this book was never put forward as containing a systematic

[1] *Transactions of the Connecticut Academy,* 1877, vol. iv. pp. 151-232.

view of Economics. It treats only of the theory, and is but an elementary sketch of elementary principles. The working out of a complete system based on these lines must be a matter of time and labour, and I know not when, if ever, I shall be able to attempt it. In the last chapter, I have, however, indicated the manner in which the theory of wages will be affected. This chapter is reprinted almost as it was written in 1871; since then the wage-fund theory has been abandoned by most English Economists, owing to the attacks of Mr. Cliffe Leslie, Mr. Shadwell, Professor Cairnes, Professor Francis Walker, and some others. Quite recently more extensive reading and more careful cogitation have led to a certain change in my ideas concerning the superstructure of Economics—in this wise:

Firstly, I am convinced that the doctrine of wages, which I adopted in 1871, under the impression that it was somewhat novel, is not really novel at all, except to those whose view is bounded by the maze of the Ricardian Economics. The true doctrine may be more or less clearly traced through the writings of a succession of great French Economists, from Condillac, Baudeau, and Le Trosne, through J.-B. Say, Destutt de Tracy, Storch, and others, down to Bastiat and Courcelle-Seneuil. The conclusion to which I am ever more clearly coming is that the only hope of attaining a true system of Economics is to fling aside, once and for ever, the mazy and preposterous assumptions of the Ricardian School. Our English

Economists have been living in a fool's paradise. The truth is with the French School, and the sooner we recognise the fact, the better it will be for all the world, except perhaps the few writers who are too far committed to the old erroneous doctrines to allow of renunciation.

Although, as I have said, the true theory of wages is not new as regards the French School, it is new, or at any rate renewed, as regards our English Schools of Economics. One of the first to treat the subject from the right point of view was Mr. Cliffe Leslie, in an article first published in *Fraser's Magazine* for July 1868, and subsequently reprinted in a collection of Essays.[1] Some years afterwards Mr. J. L. Shadwell independently worked out the same theory of wages which he has fully expounded in his admirable *System of Political Economy*.[2] In Hearn's *Plutology*, however, as pointed out in the text of this book (pp. 271-273), we find the same general idea that wages are the share of the produce which the laws of supply and demand enable the labourer to secure. It is probable that like ideas might be traced in other works were this the place to attempt a history of the subject.

Secondly, I feel sure that when, casting ourselves free from the Wage-Fund Theory, the Cost of Production doctrine of Value, the Natural Rate of

[1] "Land Systems and Industrial Economy of Ireland, England, and Continental Countries." London, 1870. Appendix, pp. 357-379.

[2] London, 1877, Trubner.

Wages, and other misleading or false Ricardian doctrines, we begin to trace out clearly and simply the results of a correct theory, it will not be difficult to arrive at a true doctrine of wages. This will probably be reached somewhat in the following way:—We must regard labour, land, knowledge, and capital as conjoint conditions of the whole produce, not as causes each of a certain portion of the produce. Thus in an elementary state of society, when each labourer owns all the three or four requisites of production, there would really be no such thing as wages, rent, or interest at all. Distribution does not arise even in idea, and the produce is simply the aggregate effect of the aggregate conditions. It is only when separate owners of the elements of production join their properties, and traffic with each other, that distribution begins, and then it is entirely subject to the principles of value and the laws of supply and demand. Each labourer must be regarded, like each landowner and each capitalist, as bringing into the common stock one part of the component elements, bargaining for the best share of the produce which the conditions of the market allow him to claim successfully. In theory the labourer has a monopoly of labour of each particular kind, as much as the landowner of land, and the capitalist of other requisite articles. Property is only another name for monopoly. But when different persons own property of exactly the same kind, they become subject to the important Law of Indifference, as I have called it (pp. 90-93), namely,

that in the same open market, at any one moment, there cannot be two prices for the same kind of article. Thus *monopoly is limited by competition*, and no owner, whether of labour, land, or capital, can, theoretically speaking, obtain a larger share of produce for it than what other owners of exactly the same kind of property are willing to accept.

So far there may seem to be nothing novel in this view; it is hardly more than will be found stated in a good many economic works. But as soon as we begin to follow out this simple view, the consequences are rather startling. We are forced, for instance, to admit that rates of wages are governed by the same formal laws as rents. This view is not new to the readers of Storch, who in the third book of his excellent *Cours d'Economie Politique* has a chapter[1] "De la Rente des talens et des qualités morales." But it is a very new doctrine to one whose economic horizon is formed by Mill and Fawcett, Ricardo and Adam Smith. Even Storch has not followed out the doctrine thoroughly; for he applies the idea of rent only to cases of *eminent* talent. It must be evident, however, that talent and capacity of all kinds are only a question of degree, so that, according to the Law of Continuity, the same principle must apply to all labourers.

A still more startling result is that, so far as cost of production regulates the values of commodities, wages must enter into the calculation on exactly the

[1] Chap. v. vol. i. p. 304.

same footing as rent. Now it is a prime point of the Ricardian doctrines that rent does not enter into cost of production. As J. S. Mill says,[1] "Rent, therefore, forms no part of the cost of production which determines the value of agricultural produce." And again,[2] "Rent is not an element in the cost of production of the commodity which yields it; except in the cases," etc. Rent in fact is represented as the effect not the cause of high value; wages on the contrary are treated as the cause, not the effect. But if rent and wages be really phenomena subject to the same formal laws, this opposite relation to value must involve error. The way out of the difficulty is furnished by the second sentence of the paragraph from which the last quotation was taken. Mill goes on to say: "But when land capable of yielding rent in agriculture is applied to some other purpose, the rent which it would have yielded is an element in the cost of production of the commodity which it is employed to produce." Here Mill edges in as an exceptional case that which proves to be the rule, reminding one of other exceptional cases described as "Some peculiar cases of value" (see p. 197 below), which I have shown to include almost all commodities.

Now Mill allows that when land capable of yielding rent in agriculture is applied to some other purpose, the rent which would have been produced

[1] *Principles of Political Economy*, book iii. chap. v. sec. 2, paragraph 3.

[2] *Ibid.* book iii. chap. vi. sec. 1, article 9.

in agriculture is an element in the cost of production of other commodities. But wherefore this distinction between agriculture and other branches of industry? Why does not the same principle apply between two different modes of agricultural employment? If land which has been yielding £2 per acre rent as pasture be ploughed up and used for raising wheat, must not the £2 per acre be debited against the expenses of the production of wheat? Suppose that somebody introduced the beetroot culture into England with a view to making sugar; this new branch of industry could not be said to pay unless it yielded, besides all other expenses, the full rents of the lands turned from other kinds of culture. But if this be conceded, the same principle must apply generally; a potato-field should pay as well as a clover-field, and a clover-field as a turnip-field; and so on. The market prices of the produce must adjust themselves so that this shall in the long run be possible. The rotation of crops, no doubt, introduces complication into the matter, but does not modify the general reasoning. The principle which emerges is that *each portion of land should be applied to that culture or use which yields the largest total of utility, as measured by the value of the produce*; if otherwise applied there will be loss. Thus the rent of land is determined by the excess of produce in the most profitable employment.

But when the matter is fully thought out, it will be seen that exactly the same principle applies to

wages. A man who can earn six shillings a day in one employment will not turn to another kind of work unless he expects to get six shillings a day or more from it. There is no such thing as absolute cost of labour; it is all matter of comparison. Every one gets the most which he can for his exertions; some can get little or nothing, because they have not sufficient strength, knowledge, or ingenuity; others get much, because they have, comparatively speaking, a monopoly of certain powers. Each seeks the work in which his peculiar faculties are most productive of utility, as measured by what other people are willing to pay for the produce. Thus wages are clearly the effect not the cause of the value of the produce. But when labour is turned from one employment to another, the wages it would otherwise have yielded must be debited to the expenses of the new product. Thus the parallelism between the theories of rent and wages is seen to be perfect in theory, however different it may appear to be in the details of application. Precisely the same view may be applied, *mutatis mutandis*, to the rent yielded by fixed capital, and to the interest of free capital. In the last case, the Law of Indifference peculiarly applies, because free capital, loanable for a certain interval, is equally available for all branches of industry; hence, at any moment and place, the interest of such capital must be the same in all branches of trade.

I ought to say that Mill, as pointed out to me by Professor Adamson, has a remarkable section at the

end of chapter v. of Book III. of the *Principles*, in which he explains that all inequalities, artificial or natural, give rise to extra gains of the nature of Rent. This section is a very satisfactory one inasmuch as it tends to support the view on which I am now insisting, a view, however, which, when properly followed out, will overthrow many of the principal doctrines of the Ricardo-Mill Economics. Those who have studied Mill's philosophic character as long and minutely as I have done, will not for a moment suppose that the occurrence of this section in Mill's book tends to establish its consistency with other portions of the same treatise.

But of course I cannot follow out the discussion of this matter in a mere preface. The results to be expected are partly indicated in my *Primer of Political Economy*, but in that little treatise my remarks upon the Origin of Rent (p. 94), as originally printed in the first edition, were erroneous, and the section altogether needs to be rewritten. When at length a true system of Economics comes to be established, it will be seen that that able but wrong-headed man, David Ricardo, shunted the car of Economic science on to a wrong line—a line, however, on which it was further urged towards confusion by his equally able and wrong-headed admirer, John Stuart Mill. There were Economists, such as Malthus and Senior, who had a far better comprehension of the true doctrines (though not free from the Ricardian errors), but they were driven out of the field by the unity and influence of the Ricardo-

Mill school. It will be a work of labour to pick up the fragments of a shattered science and to start anew, but it is a work from which they must not shrink who wish to see any advance of Economic Science.

THE CHESTNUTS,
HAMPSTEAD HEATH, N.W.
May, 1879.

PREFACE TO THE THIRD EDITION

(1888)

THE present edition of the *Theory of Political Economy* is an exact reprint of the second edition, with the exception of the first Appendix containing the bibliographical list of mathematico-economic books. I desired to add to that list several books which it had been my husband's intention to include in the next edition, and when I consulted my friend, Mr. H. S. Foxwell, he advised me to continue it up to the present date. I am greatly indebted to the kindness of those friends who have enabled me to accomplish this; and amongst others my thanks are especially due to the Rev. P. H. Wicksteed, Professor F. Y. Edgeworth, and Professor Harald Westergaard of Copenhagen, for the trouble they have taken in revising proofs for me, as well as in supplying me with the titles of those books which ought to be included. We have endeavoured to follow the rules which Mr. Jevons has laid down in the preface to the second edition, and though the list is probably not

complete, I hope that no work of importance has been omitted.

A few books published during my husband's lifetime, but which were, I believe, unknown to him, are now included, but I have enclosed them within brackets. I have also marked the place at which the additional list prepared by himself ends.

HARRIET A. JEVONS.

August, 1888.

PREFACE TO THE FOURTH EDITION

In preparing a new edition of this work, I had to keep in mind two facts: first, that it is still read by a large number of students for the sake of its exposition of some of the fundamental principles of economics which are not as yet more adequately treated in any other book in the English language; and secondly, that it is already in some sense a classic. An occasional alteration of the text, and some explanation of difficult passages, seemed desirable for the assistance of the student; but the interest of the book as the first important English attempt to develop a system of economics from a basis of psychological facts, and by the mathematical method, prescribed a faithful reprint of the text as the author himself left it. I have adhered so far to the latter course, that, with the exception of a few slight changes of punctuation, designed more readily to exhibit the sense, there are practically no alterations of the text. The very few which I have permitted myself are such as were needed to correct obvious slips, save one (p. 263, line 1) which was indicated by the author himself in his copy of the second edition. On the other

hand, wherever a short footnote could render the author's meaning clearer, or could correct one of the few slight errors which I have discovered, I have not hesitated to insert it, always distinguishing the new footnotes with the mark—[Ed.].

The appendixes I have felt at liberty to treat more freely than the text, and they now embrace three new ones. The first appendix is an attempt to elucidate my father's treatment of the theory of interest, the truth and originality of which have not, I think, been widely enough appreciated, owing to the perplexity caused by his omission to make his assumptions clear and to interpret his result in concrete terms. I can only hope that my view of his meaning will be endorsed by others who may look into his statement of the theory once more.

In Appendix II. I have printed a fragment of MS. on Capital and Interest which I found amongst my father's notes intended for his *Principles of Economics*. Somehow it escaped publication in the volume devoted to the fragments of the Principles, but it seemed to me sufficiently original and complete to be worth preserving in permanent form.

Appendix III. is inserted for its historic interest, being a reprint of the author's first statement of his newly devised mathematical theory of economics. He read this paper at the meeting of the British Association at Cambridge in 1862, and an abstract of it was printed in the Report of the Association for that year. The paper was afterwards printed in full

in the *Journal of the Statistical Society* for 1866; but both publications failed to attract attention. According to one of my father's MS. notes, which I have been lucky enough to discover, the publication of the *Theory of Political Economy* might have been delayed considerably later than 1871 had it not been for the appearance in 1868 and 1870 of articles by Professor Fleeming Jenkin (see Appendix V.) which are distinctly mathematical in method and contain a number of very ingenious geometrical diagrams illustrating the laws of supply and demand. My father in this note mentions that Professor Jenkin had opened a correspondence with him on the subject of the mathematical treatment of economics and the use of curves, and that this was followed by the publication of the latter's "Graphic Representation of the Laws of Supply and Demand," and concludes: "Partly in consequence of this I was led to write and publish the *Theory* in 1871."

Appendix IV. contains the author's bibliography of his own writings on economic subjects, which has been amplified and brought up to date to include recent new editions of his works.

The fifth Appendix contains Professor Jevons's own list of mathematico-economic writings, with the addition of a few books unknown to him which are enclosed in square brackets. It will be noticed that it has been enlarged by valuable notes on the contents of many of the books. The MS. of these notes in my father's hand I found in a box with letters and other

notes referring to the mathematical treatment of economics. Whether they were prepared merely for his own use in writing the preface to the second edition; or whether, having intended to publish them with his bibliography in the second edition, he rejected them as too lengthy; or whether he was waiting to complete his notes for a third edition, or some separate publication, I do not know. They are evidence of the enthusiasm and labour which he devoted to the subject, and can hardly fail to be of use to future students to whom access to the original papers may often be difficult.

It was my intention to continue my father's work by drawing up a bibliography of all books, articles, and published writings of any kind which have appeared from 1880 to 1910 inclusive, so far as I was able to discover them, to form a sixth appendix. Living as I do without access to any good library, the work proceeded slowly; and the decision to produce a classified bibliography, which entailed reading a good part of many papers, further retarded matters. Mathematical works on economics have happily become so numerous that a mere list of them without indication of their contents would be of little use.

For convenience in searching periodicals, I adopted the rule of including all papers dealing with economic theory by the graphic or symbolic methods, and those only; but a few other books and articles not employing any mathematical mode of expression were to be

included on the ground that their subject matter is dealt with in a manner amply recognising the quantitative character of the science. Although several hundreds of card entries have been made and classified, the work is still very far from complete. As there seems little prospect of my being able to give time to its completion in the near future, I have at last reluctantly decided to issue this edition without this general bibliography.

Probably this is a wise decision from several points of view. It will not only allow of its being published later as an independent book in a handy form, but I believe that by the co-operation of other economists, which I intend soon to seek, a bibliography might ultimately emerge which might be of the greatest service to the progress of this science.

If a dedication were not somewhat out of place in a volume which is only a new edition by another than the author, I should like by such a sign to associate my mother closely with the reissue of this book. She watched with lively interest and sympathy the issue of the first two editions; the third she herself prepared. I remember as a child reading the proofs aloud to her, and she followed with eager interest my preparation of this fourth edition. The book is so much hers that I wish she could have lived to have the satisfaction of seeing the issue of what is probably the final edition.

CONTENTS

CHAPTER IV

THEORY OF EXCHANGE

CHAPTER V

THEORY OF LABOUR

CHAPTER VI

THEORY OF RENT

CHAPTER VII

THEORY OF CAPITAL

CHAPTER VIII

CONCLUDING REMARKS

APPENDIX I

APPENDIX II

APPENDIX III

APPENDIX IV

APPENDIX V

CHAPTER I

INTRODUCTION

THE science of Political Economy rests upon a few notions of an apparently simple character. Utility, wealth, value, commodity, labour, land, capital, are the elements of the subject; and whoever has a thorough comprehension of their nature must possess or be soon able to acquire a knowledge of the whole science. As almost every economic writer has remarked, it is in treating the simple elements that we require the most care and precision, since the least error of conception must vitiate all our deductions. Accordingly, I have devoted the following pages to an investigation of the conditions and relations of the above-named notions.

Repeated reflection and inquiry have led me to the somewhat novel opinion, that *value depends entirely upon utility.* Prevailing opinions make labour rather than utility the origin of value; and there are even those who distinctly assert that labour is the *cause* of value. I show, on the contrary, that we have only to trace out carefully the natural laws

of the variation of utility, as depending upon the quantity of commodity in our possession, in order to arrive at a satisfactory theory of exchange, of which the ordinary laws of supply and demand are a necessary consequence. This theory is in harmony with facts; and, whenever there is any apparent reason for the belief that labour is the cause of value, we obtain an explanation of the reason. Labour is found often to determine value, but only in an indirect manner, by varying the degree of utility of the commodity through an increase or limitation of the supply.

These views are not put forward in a hasty or ill-considered manner. All the chief points of the theory were sketched out ten years ago; but they were then published only in the form of a brief paper communicated to the Statistical or Economic Section of the British Association at the Cambridge Meeting, which took place in the year 1862. A still briefer abstract of that paper was inserted in the Report of the Meeting,[1] and the paper itself was not printed until June 1866.[2] Since writing that paper, I have, over and over again, questioned the truth of my own notions, but without ever finding any reason to doubt their substantial correctness.

[1] *Reports of Sections*, p. 158.

[2] *Journal of the Statistical Society*, vol. xxix. p. 282.

Mathematical Character of the Science

It is clear that Economics, if it is to be a science at all, must be a mathematical science. There exists much prejudice against attempts to introduce the methods and language of mathematics into any branch of the moral sciences. Many persons seem to think that the physical sciences form the proper sphere of mathematical method, and that the moral sciences demand some other method,—I know not what. My theory of Economics, however, is purely mathematical in character. Nay, believing that the quantities with which we deal must be subject to continuous variation, I do not hesitate to use the appropriate branch of mathematical science, involving though it does the fearless consideration of infinitely small quantities. The theory consists in applying the differential calculus to the familiar notions of wealth, utility, value, demand, supply, capital, interest, labour, and all the other quantitative notions belonging to the daily operations of industry. As the complete theory of almost every other science involves the use of that calculus, so we cannot have a true theory of Economics without its aid.

To me it seems that *our science must be mathematical, simply because it deals with quantities.* Wherever the things treated are capable of being *greater or less*, there the laws and relations must be mathematical in nature. The ordinary laws of supply

and demand treat entirely of quantities of commodity demanded or supplied, and express the manner in which the quantities vary in connection with the price. In consequence of this fact the laws *are* mathematical. Economists cannot alter their nature by denying them the name; they might as well try to alter red light by calling it blue. Whether the mathematical laws of Economics are stated in words, or in the usual symbols, x, y, z, p, q, etc., is an accident, or a matter of mere convenience. If we had no regard to trouble and prolixity, the most complicated mathematical problems might be stated in ordinary language, and their solution might be traced out by words. In fact, some distinguished mathematicians have shown a liking for getting rid of their symbols, and expressing their arguments and results in language as nearly as possible approximating to that in common use. In his *Système du Monde*, Laplace attempted to describe the truths of physical astronomy in common language; and Thomson and Tait interweave their great *Treatise on Natural Philosophy* with an interpretation in ordinary words, supposed to be within the comprehension of general readers.[1]

[1] The large-type or non-symbolic portion of the Treatise has been reprinted in a separate volume, under the title *Elements of Natural Philosophy*, by Professors Sir W. Thomson and P. G. Tait. Part I. Oxford, Clarendon Press, 1873. But the authors appear to me to be inaccurate in describing this work, in the preface, as *non-mathematical*. It is comparatively *non-symbolic*, but equally mathematical with the complete Treatise.

These attempts, however distinguished and ingenious their authors, soon disclose the inherent defects of the grammar and dictionary for expressing complicated relations. The symbols of mathematical books are not different in nature from language; they form a perfected system of language, adapted to the notions and relations which we need to express. They do not constitute the mode of reasoning they embody; they merely facilitate its exhibition and comprehension. If, then, in Economics, we have to deal with quantities and complicated relations of quantities, we must reason mathematically; we do not render the science less mathematical by avoiding the symbols of algebra, —we merely refuse to employ, in a very imperfect science, much needing every kind of assistance, that apparatus of appropriate signs which is found indispensable in other sciences.

Confusion between Mathematical and Exact Sciences

Many persons entertain a prejudice against mathematical language, arising out of a confusion between the ideas of a mathematical science and an exact science. They think that we must not pretend to calculate unless we have the precise data which will enable us to obtain a precise answer to our calculations; but, in reality, there is no such thing as an exact science, except in a comparative sense. Astronomy is more exact than other sciences, because the

position of a planet or star admits of close measurement; but, if we examine the methods of physical astronomy, we find that they are all approximate. Every solution involves hypotheses which are not really true: as, for instance, that the earth is a smooth, homogeneous spheroid. Even the apparently simpler problems in statics or dynamics are only hypothetical approximations to the truth.[1]

We can calculate the effect of a crowbar, provided it be perfectly inflexible and have a perfectly hard fulcrum,—which is never the case.[2] The data are almost wholly deficient for the complete solution of any one problem in natural science. Had physicists waited until their data were perfectly precise before they brought in the aid of mathematics, we should have still been in the age of science which terminated at the time of Galileo.

When we examine the less precise physical sciences, we find that physicists are, of all men, most bold in developing their mathematical theories in advance of their data. Let any one who doubts this examine Airy's "Theory of the Tides," as given in the *Encyclopædia Metropolitana*; he will there find a wonderfully complex mathematical theory which is confessed by its author to be incapable of exact or even approximate application, because the results of

[1] This subject of the approximate character of quantitative science is pursued, at some length, in my *Principles of Science*, chap. xxi., on "The Theory of Approximation," and elsewhere in the same work.

[2] Thomson and Tait's *Treatise on Natural Philosophy*, vol. i. p. 337.

the various and often unknown contours of the seas do not admit of numerical verification. In this and many other cases we have mathematical theory without the data requisite for precise calculation.

The greater or less accuracy attainable in a mathematical science is a matter of accident, and does not affect the fundamental character of the science. There can be but two classes of sciences—those which are *simply logical*, and *those which, besides being logical, are also mathematical.* If there be any science which determines merely whether a thing be or be not—whether an event will happen, or will not happen—it must be a purely logical science; but if the thing may be greater or less, or the event may happen sooner or later, nearer or farther, then quantitative notions enter, and the science must be mathematical in nature, by whatever name we call it.

Capability of Exact Measurement

Many will object, no doubt, that the notions which we treat in this science are incapable of any measurement. We cannot weigh, nor gauge, nor test the feelings of the mind; there is no unit of labour, or suffering, or enjoyment. It might thus seem as if a mathematical theory of Economics would be necessarily deprived for ever of numerical data.

I answer, in the first place, that nothing is less warranted in science than an uninquiring and unhoping spirit. In matters of this kind, those who

despair are almost invariably those who have never tried to succeed. A man might be despondent had he spent a lifetime on a difficult task without a gleam of encouragement; but the popular opinions on the extension of mathematical theory tend to deter any man from attempting tasks which, however difficult, ought, some day, to be achieved.

If we trace the history of other sciences, we gather no lessons of discouragement. In the case of almost everything which is now exactly measured, we can go back to the age when the vaguest notions prevailed. Previous to the time of Pascal, who would have thought of measuring *doubt* and *belief*? Who could have conceived that the investigation of petty games of chance would have led to the creation of perhaps the most sublime branch of mathematical science—the theory of probabilities? There are sciences which, even within the memory of men now living, have become exactly quantitative. While Quesnay and Baudeau and Le Trosne and Condillac were founding Political Economy in France, and Adam Smith in England, electricity was a vague phenomenon, which was known, indeed, to be capable of becoming greater or less, but was not measured nor calculated: it is within the last forty or fifty years that a mathematical theory of electricity, founded on exact data, has been established. We now enjoy precise quantitative notions concerning heat, and can measure the temperature of a body to less than $\frac{1}{5000}$ part of a degree centigrade. Compare this precision with that

of the earliest makers of thermometers, the Academicians del Cimento, who used to graduate their instruments by placing them in the sun's rays to obtain a point of fixed temperature.[1]

De Morgan excellently said,[2] "As to some magnitudes, the clear idea of measurement comes soon: in the case of length, for example. But let us take a more difficult one, and trace the steps by which we acquire and fix the idea: say *weight*. What weight is, we need not know. . . . We know it as a magnitude before we give it a name: any child can discover the *more* that there is in a bullet, and the less that there is in a cork of twice its size. Had it not been for the simple contrivance of the balance, which we are well assured (how, it matters not here) enables us to poise equal weights against one another, that is, to detect equality and inequality, and thence to ascertain how many times the greater contains the less, we might not to this day have had much clearer ideas on the subject of weight, as a magnitude, than we have on those of talent, prudence, or self-denial, looked at in the same light. All who are ever so little of geometers will remember the time when their notions of an angle, as a magnitude, were as vague as, perhaps more so than, those of a moral quality; and they will also remember the steps by which this vagueness became clearness and precision."

Now there can be no doubt that pleasure, pain,

[1] See *Principles of Science*, chap. xiii., on "The Exact Measurement of Phenomena," 3d ed. p. 270.

[2] *Formal Logic*, p. 175.

labour, utility, value, wealth, money, capital, etc., are all notions admitting of quantity; nay, the whole of our actions in industry and trade certainly depend upon comparing quantities of advantage or disadvantage. Even the theories of moralists have recognised the quantitative character of the subject. Bentham's *Introduction to the Principles of Morals and Legislation* is thoroughly mathematical in the character of the method. He tells us[1] to estimate the tendency of an action thus: "Sum up all the values of all the pleasures on the one side, and those of all the pains on the other. The balance, if it be on the side of pleasure, will give the good tendency of the act upon the whole, with respect to the interests of that individual person; if on the side of pain, the bad tendency of it upon the whole." The mathematical character of Bentham's treatment of moral science is also well exemplified in his remarkable tract entitled, "A Table of the Springs of Action," printed in 1817, as in p. 3, and elsewhere.

"But where," the reader will perhaps ask, "are your numerical data for estimating pleasures and pains in Political Economy?" I answer, that my numerical data are more abundant and precise than those possessed by any other science, but that we have not yet known how to employ them. The very abundance of our data is perplexing. There is not a clerk nor book-keeper in the country who is not

[1] Chapter iv., on the "Value of a Lot of Pleasure or Pain, How to be measured," sec. v. 5.

engaged in recording numerical facts for the economist. The private-account books, the great ledgers of merchants and bankers and public offices, the share lists, price lists, bank returns, monetary intelligence, Custom-house and other Government returns, are all full of the kind of numerical data required to render Economics an exact mathematical science. Thousands of folio volumes of statistical, parliamentary, or other publications await the labour of the investigator. It is partly the very extent and complexity of the information which deters us from its proper use. But it is chiefly a want of method and completeness in this vast mass of information which prevents our employing it in the scientific investigation of the natural laws of Economics.

I hesitate to say that men will ever have the means of measuring directly the feelings of the human heart. A unit of pleasure or of pain is difficult even to conceive; but it is the amount of these feelings which is continually prompting us to buying and selling, borrowing and lending, labouring and resting, producing and consuming; and *it is from the quantitative effects of the feelings that we must estimate their comparative amounts.* We can no more know nor measure gravity in its own nature than we can measure a feeling; but, just as we measure gravity by its effects in the motion of a pendulum, so we may estimate the equality or inequality of feelings by the decisions of the human mind. The will is our pendulum, and its oscillations

are minutely registered in the price lists of the markets. I know not when we shall have a perfect system of statistics, but the want of it is the only insuperable obstacle in the way of making Economics an exact science. In the absence of complete statistics, the science will not be less mathematical, though it will be immensely less useful than if it were, comparatively speaking, exact. A correct theory is the first step towards improvement, by showing what we need and what we might accomplish.

Measurement of Feeling and Motives

Many readers may, even after reading the preceding remarks, consider it quite impossible to create such a calculus as is here contemplated, because we have no means of defining and measuring quantities of feeling, like we can measure a mile, or a right angle, or any other physical quantity. I have granted that we can hardly form the conception of a unit of pleasure or pain, so that the numerical expression of quantities of feeling seems to be out of the question. But we only employ units of measurement in other things to facilitate the comparison of quantities; and if we can compare the quantities directly, we do not need the units. Now the mind of an individual is the balance which makes its own comparisons, and is the final judge of quantities of feeling. As Mr. Bain says,[1] "It is only an identical proposition to affirm

[1] *The Emotions and the Will*, 1st ed., p. 447.

that the greatest of two pleasures, or what appears such, sways the resulting action; for it is this resulting action that alone determines which is the greater."

Pleasures, in short, are, for the time being, as the mind estimates them; so that we cannot make a choice, or manifest the will in any way, without indicating thereby an excess of pleasure in some direction. It is true that the mind often hesitates and is perplexed in making a choice of great importance: this indicates either varying estimates of the motives, or a feeling of incapacity to grasp the quantities concerned. I should not think of claiming for the mind any accurate power of measuring and adding and subtracting feelings, so as to get an exact balance. We can seldom or never affirm that one pleasure is an exact multiple of another; but the reader who carefully criticises the following theory will find that it seldom involves the comparison of quantities of feeling differing much in amount. The theory turns upon those critical points where pleasures are nearly, if not quite, equal. I never attempt to estimate the whole pleasure gained by purchasing a commodity; the theory merely expresses that, when a man has purchased enough, he would derive equal pleasure from the possession of a small quantity more as he would from the money price of it. Similarly, the whole amount of pleasure that a man gains by a day's labour hardly enters into the question; it is when a man is doubtful whether to increase his

hours of labour or not, that we discover an equality between the pain of that extension and the pleasure of the increase of possessions derived from it.

The reader will find, again, that there is never, in any single instance, an attempt made to compare the amount of feeling in one mind with that in another. I see no means by which such comparison can be accomplished. The susceptibility of one mind may, for what we know, be a thousand times greater than that of another. But, provided that the susceptibility was different in a like ratio in all directions, we should never be able to discover the difference. Every mind is thus inscrutable to every other mind, and no common denominator of feeling seems to be possible. But even if we could compare the feelings of different minds, we should not need to do so; for one mind only affects another indirectly. Every event in the outward world is represented in the mind by a corresponding motive, and it is by the balance of these that the will is swayed. But the motive in one mind is weighed only against other motives in the same mind, never against the motives in other minds. Each person is to other persons a portion of the outward world—the *non-ego* as the metaphysicians call it. Thus motives in the mind of A may give rise to phenomena which may be represented by motives in the mind of B; but between A and B there is a gulf. Hence the weighing of motives must always be confined to the bosom of the individual.

I must here point out that, though the theory

presumes to investigate the condition of a mind, and bases upon this investigation the whole of Economics, practically it is an aggregate of individuals which will be treated. The general forms of the laws of Economics are the same in the case of individuals and nations; and, in reality, it is a law operating in the case of multitudes of individuals which gives rise to the aggregate represented in the transactions of a nation. Practically, however, it is quite impossible to detect the operation of general laws of this kind in the actions of one or a few individuals. The motives and conditions are so numerous and complicated, that the resulting actions have the appearance of caprice, and are beyond the analytic powers of science. With every increase in the price of such a commodity as sugar, we ought, theoretically speaking, to find every person reducing his consumption by a small amount, and according to some regular law. In reality, many persons would make no change at all; a few, probably, would go to the extent of dispensing with the use of sugar altogether so long as its cost continued to be excessive. It would be by examining the average consumption of sugar in a large population that we might detect a continuous variation, connected with the variation of price by a constant law. It would not, of necessity, happen that the law would be exactly the same in the case of aggregates and individuals, unless all those individuals were of the same character and position as regards wealth and habits; but there would be a more or less regular

law to which the same kind of formulæ would apply. The use of an average, or, what is the same, an aggregate result, depends upon the high probability that accidental and disturbing causes will operate, in the long run, as often in one direction as the other, so as to neutralise each other. Provided that we have a sufficient number of independent cases, we may then detect the effect of any *tendency*, however slight. Accordingly, questions which appear, and perhaps are, quite indeterminate as regards individuals, may be capable of exact investigation and solution in regard to great masses and wide averages.[1]

Logical Method of Economics

The logical method of Economics as a branch of the social sciences is a subject on which much might be written, and on which very diverse opinions are held at the present day (1879). In this place I can only make a few brief remarks. I think that John Stuart Mill is substantially correct in considering our science to be a case of what he calls[2] the Physical or Concrete Deductive Method; he considers that we may start from some obvious psychological law, as for instance, that a greater gain is preferred to a smaller one, and we may then reason downwards, and predict the phenomena which will be produced in society by

[1] Concerning the meaning and employment of *Averages*, see *Principles of Science*, chap. xvi., on "The Method of Means."

[2] *System of Logic*, book vi., chap. ix. sec. 3.

such a law. The causes in action in any community are, indeed, so complicated that we shall seldom be able to discover the undisturbed effects of any one law, but, so far as we can analyse the statistical phenomena observed, we obtain a verification of our reasoning. This view of the matter is almost identical with that adopted by the late Professor Cairnes in his lectures on "The Character and Logical Method of Political Economy."[1]

The principal objection to be urged against this treatment of the subject, is that Mill has described the Concrete Deductive Method as if it were one of many inductive methods. In my Elementary Lessons in Logic (p. 258), I proposed to call the method the *Complete Method*, as implying that it combines observation, deduction, and induction in the most complete and perfect way. But I subsequently arrived at the conclusion that this so-called Deductive Method is no special method at all, but simply induction itself in its essential form. As I have fully explained,[2] Induction is an *inverse operation*, the inverse of Deduction, and can only be performed by the use of deduction. Possessing certain facts of observation, we frame an hypothesis as to the laws governing those facts; we reason from the hypothesis deductively to the results to be expected; and we then examine these results in connection with the facts in question; coincidence confirms the whole reasoning; conflict

[1] 2nd ed. (Macmillan), 1875.

[2] *Principles of Science*, chaps. vii., ix., xii., etc.

obliges us either to seek for disturbing causes, or else to abandon our hypothesis. In this procedure there is nothing peculiar; when properly understood it is found to be the method of all the inductive sciences.

The science of Economics, however, is in some degree peculiar, owing to the fact, pointed out by J. S. Mill and Cairnes, that its ultimate laws are known to us immediately by intuition, or, at any rate, they are furnished to us ready made by other mental or physical sciences. That every person will choose the greater apparent good; that human wants are more or less quickly satiated; that prolonged labour becomes more and more painful, are a few of the simple inductions on which we can proceed to reason deductively with great confidence. From these axioms we can deduce the laws of supply and demand, the laws of that difficult conception, value, and all the intricate results of commerce, so far as data are available. The final agreement of our inferences with *à posteriori* observations ratifies our method. But unfortunately this verification is often the least satisfactory part of the process, because, as J. S. Mill has fully explained, the circumstances of a nation are infinitely complicated, and we seldom get two or more instances which are comparable. To fulfil the conditions of inductive inquiry, we ought to be able to observe the effects of a cause coming singly into action, while all other causes remain unaltered. Entirely to prove the good effects of Free Trade in

England, for example, we ought to have the nation unaltered in every circumstance except the abolition of burdens and restrictions on trade.[1] But it is obvious that while Free Trade was being introduced into England, many other causes of prosperity were also coming into action—the progress of invention, the construction of railways, the profuse consumption of coal, the extension of the colonies, etc. etc. Although, then, the beneficent results of Free Trade are great and unquestionable, they could hardly be proved to exist *à posteriori*; they are to be believed because deductive reasoning from premises of almost certain truth leads us confidently to expect such results, and there is nothing in experience which in the least conflicts with our expectations. In spite of occasional revulsions, due to periodical fluctuations depending on physical causes, the immense prosperity of the country since the adoption of Free Trade confirms our anticipations as far as, under complex circumstances, facts are capable of doing so. It will thus be seen that Political Economy tends to be more deductive than many of the physical sciences, in which closely approximate verification is often possible; but, even so far as the science is inductive, it involves the use of deductive reasoning, as already explained.

Within the last year or two, much discussion has been raised concerning the Philosophical Method of Political Economy, by Mr. T. E. Cliffe Leslie's

[1] *Principles of Science*, chap. xix., on "Experiment."

interesting Essay on that subject,[1] as also by the recent address of Dr. Ingram at the Dublin Meeting of the British Association.[2] I quite concur with these able and eminent economists so far as to allow that historical investigation is of great importance in Social Science. But, instead of converting our present science of economics into an historical science, utterly destroying it in the process, I would perfect and develop what we already possess, and at the same time erect a new branch of social science on an historical foundation. This new branch of science, on which many learned men, such as Richard Jones, De Laveleye, Lavergne, Cliffe Leslie, Sir Henry Maine, Thorold Rogers, have already laboured, is doubtless a portion of what Herbert Spencer calls Sociology, the Science of the Evolution of Social Relations. Political Economy is in a chaotic state at present, because there is need of subdividing a too extensive sphere of knowledge. Quesnay, Sir James Steuart, Baudeau, Le Trosne, and Condillac first differentiated Economics sufficiently to lead it to be regarded as a distinct science; it has since been loaded with great accretions due to the progress of investigation. It is only by subdivision, by recognising a branch of Economic Sociology, together possibly with two or three other branches of statistical, jural, or social science, that we can rescue our science from its

[1] *Hermathena*, No. iv., Dublin, 1876, p. 1.

[2] *Statistical Journal*, January 1879, vol. xli. p. 602. Also reprint by Longmans, 1878.

confused state. I have already endeavoured to show the need of this step in a lecture delivered at the University College, in October 1876,[1] and I shall perhaps have a future opportunity of enlarging more upon the subject.

To return, however, to the topic of the present work, the theory here given may be described as *the mechanics of utility and self-interest.* Oversights may have been committed in tracing out its details, but in its main features this theory must be the true one. Its method is as sure and demonstrative as that of kinematics or statics, nay, almost as self-evident as are the elements of Euclid, when the real meaning of the formulæ is fully seized.

I do not hesitate to say, too, that Economics might be gradually erected into an exact science, if only commercial statistics were far more complete and accurate than they are at present, so that the formulæ could be endowed with exact meaning by the aid of numerical data. These data would consist chiefly in accurate accounts of the quantities of goods possessed and consumed by the community, and the prices at which they are exchanged. There is no reason whatever why we should not have those statistics, except the cost and trouble of collecting them, and the unwillingness of persons to afford information. The quantities themselves to be measured and registered are most concrete and precise. In a few

[1] *Fortnightly Review*, Dec. 1876; "The Future of Political Economy." [Reprinted in the author's *Principles of Economics* (1905).]

cases we already have information approximating to completeness, as when a commodity like tea, sugar, coffee, or tobacco is wholly imported. But when articles are untaxed, and partly produced within the country, we have yet the vaguest notions of the quantities consumed. Some slight success is now, at last, attending the efforts to gather agricultural statistics; and the great need felt by men engaged in the cotton and other trades to obtain accurate accounts of stocks, imports, and consumption, will probably lead to the publication of far more complete information than we have hitherto enjoyed.

The deductive science of Economics must be verified and rendered useful by the purely empirical science of Statistics. Theory must be invested with the reality and life of fact. But the difficulties of this union are immensely great, and I appreciate them quite as much as does Cairnes in his admirable lectures "On the Character and Logical Method of Political Economy." I make hardly any attempt to employ statistics in this work, and thus I do not pretend to any numerical precision. But, before we attempt any investigation of facts, we must have correct theoretical notions; and of what are here presented, I would say, in the words of Hume, in his *Essay on Commerce*, "If false, let them be rejected: but no one has a right to entertain a prejudice against them merely because they are out of the common road."

Relation of Economics to Ethics

I wish to say a few words, in this place, upon the relation of Economics to Moral Science. The theory which follows is entirely based on a calculus of pleasure and pain; and the object of Economics is to maximise happiness by purchasing pleasure, as it were, at the lowest cost of pain. The language employed may be open to misapprehension, and it may seem as if pleasures and pains of a gross kind were treated as the all-sufficient motives to guide the mind of man. I have no hesitation in accepting the Utilitarian theory of morals which does uphold the effect upon the happiness of mankind as the criterion of what is right and wrong. But I have never felt that there is anything in that theory to prevent our putting the widest and highest interpretation upon the terms used.

Jeremy Bentham put forward the Utilitarian theory in the most uncompromising manner. According to him, whatever is of interest or importance to us must be the cause of pleasure or of pain; and when the terms are used with a sufficiently wide meaning, pleasure and pain include all the forces which drive us to action. They are explicitly or implicitly the matter of all our calculations, and form the ultimate quantities to be treated in all the moral sciences. The words of Bentham on this subject may require some explanation and qualification, but they

are too grand and too full of truth to be omitted. "Nature," he says,[1] "has placed mankind under the governance of two sovereign masters — *pain* and *pleasure*. It is for them alone to point out what we ought to do, as well as to determine what we shall do. On the one hand the standard of right and wrong, on the other the chain of causes and effects, are fastened to their throne. They govern us in all we do, in all we say, in all we think: every effort we can make to throw off our subjection will serve but to demonstrate and confirm it. In words a man may pretend to abjure their empire; but, in reality, he will remain subject to it all the while. The *principle of utility* recognises this subjection, and assumes it for the foundation of that system, the object of which is to rear the fabric of felicity by the hands of reason and of law. Systems which attempt to question it deal in sounds instead of sense, in caprice instead of reason, in darkness instead of light."

In connection with this passage we may take that of Paley, who says, with his usual clear brevity,[2] "I hold that pleasures differ in nothing but in continuance and intensity."

The acceptance or non-acceptance of the basis of the Utilitarian doctrine depends, in my mind, on the exact interpretation of the language used. As it seems to me, the feelings of which a man is capable

[1] *An Introduction to the Principles of Morals and Legislation*, by Jeremy Bentham. Edition of 1823, vol. i. p. 1.

[2] *Principles of Moral and Political Philosophy*, book i., chap. vi.

are of various grades. He is always subject to mere physical pleasure or pain, necessarily arising from his bodily wants and susceptibilities. He is capable also of mental and moral feelings of several degrees of elevation. A higher motive may rightly overbalance all considerations belonging even to the next lower range of feelings; but so long as the higher motive does not intervene, it is surely both desirable and right that the lower motives should be balanced against each other. Starting with the lowest stage —it is a man's duty, as it is his natural inclination, to earn sufficient food and whatever else may best satisfy his proper and moderate desires. If the claims of a family or of friends fall upon him, it may become desirable that he should deny his own desires and even his physical needs their full customary gratification. But the claims of a family are only a step to a higher grade of duties.

The safety of a nation, the welfare of great populations, may happen to depend upon his exertions, if he be a soldier or a statesman: claims of a very strong kind may now be overbalanced by claims of a still stronger kind. Nor should I venture to say that, at any point, we have reached the highest rank —the supreme motives which should guide the mind. The statesman may discover a conflict between motives; a measure may promise, as it would seem, the greatest good to great numbers, and yet there may be motives of uprightness and honour that may hinder his promoting the measure. How such

difficult questions may be rightly determined it is not my purpose to inquire here.

The utilitarian theory holds, that all forces influencing the mind of man are pleasures and pains; and Paley went so far as to say that all pleasures and pains are of one kind only. Mr. Bain has carried out this view to its complete extent, saying,[1] "No amount of complication is ever able to disguise the general fact, that our voluntary activity is moved by only two great classes of stimulants; either a pleasure or a pain, present or remote, must lurk in every situation that drives us into action." The question certainly appears to turn upon the language used. Call any motive which attracts us to a certain course of conduct, pleasure; and call any motive which deters us from that conduct, pain; and it becomes impossible to deny that all actions are governed by pleasure and pain. But it then becomes indispensable to admit that a single higher pleasure will sometimes neutralise a vast extent and continuance of lower pains. It seems hardly possible to admit Paley's statement, except with an interpretation that would probably reverse his intended meaning. Motives and feelings are certainly of the same kind to the extent that we are able to weigh them against each other; but they are, nevertheless, almost incomparable in power and authority.

My present purpose is accomplished in pointing out this hierarchy of feeling, and assigning a proper

[1] *The Emotions and the Will*, 1st ed., p. 460.

place to the pleasures and pains with which the Economist deals. It is the lowest rank of feelings which we here treat. The calculus of utility aims at supplying the ordinary wants of man at the least cost of labour. Each labourer, in the absence of other motives, is supposed to devote his energy to the accumulation of wealth. A higher calculus of moral right and wrong would be needed to show how he may best employ that wealth for the good of others as well as himself. But when that higher calculus gives no prohibition, we need the lower calculus to gain us the utmost good in matters of moral indifference. There is no rule of morals to forbid our making two blades of grass grow instead of one, if, by the wise expenditure of labour, we can do so. And we may certainly say, with Francis Bacon, "while philosophers are disputing whether virtue or pleasure be the proper aim of life, do you provide yourself with the instruments of either."

CHAPTER II

THEORY OF PLEASURE AND PAIN

Pleasure and Pain as Quantities

PROCEEDING to consider how pleasure and pain can be estimated as magnitudes, we must undoubtedly accept what Bentham has laid down upon this subject. "To a person," he says,[1] "considered *by himself*, the value of a pleasure or pain, considered *by itself*, will be greater or less according to the four following circumstances:—

(1) Its *intensity.*
(2) Its *duration.*
(3) Its *certainty* or *uncertainty.*
(4) Its *propinquity* or *remoteness.*

These are the circumstances which are to be considered in estimating a pleasure or a pain considered each of them by itself."

[1] *An Introduction to the Principles of Morals and Legislation*, 2nd ed., 1823, vol. i. p. 49. The earliest writer who, so far as I know, has treated Pleasure and Pain in a definitely quantitative manner, is Francis Hutcheson, in his *Essay on the Nature and Conduct of the Passions and Affections*, 1728, pp. 34-43, 126, etc.

Bentham[1] goes on to consider three other circumstances which relate to the ultimate and complete result of any act or feeling; these are—

(5) *Fecundity*, or the chance a feeling has of being followed by feelings of the same kind: that is, pleasures, if it be a pleasure; pains, if it be a pain.
(6) *Purity*, or the chance it has of not being followed by feelings of an opposite kind. And
(7) *Extent*, or the number of persons to whom it extends, and who are affected by it.

These three last circumstances are of high importance as regards the theory of morals; but they will not enter into the more simple and restricted problem which we attempt to solve in Economics.

A feeling, whether of pleasure or of pain, must be regarded as having two dimensions, or modes of varying in regard to quantity. Every feeling must last some time, and it may last a longer or shorter time; while it lasts, it may be more or less acute and intense. If in two cases the duration of feeling is the same, that case will produce the greater quantity which is the more intense; or we may say that, with the same duration, the quantity will be proportional to the intensity. On the other hand, if the intensity of a feeling were to remain constant, the quantity of feeling would increase with its duration. Two days of the same degree of happiness

[1] Introduction, p. 50.

are to be twice as much desired as one day; two days of suffering are to be twice as much feared. If the intensity ever continued fixed, the whole quantity would be found by multiplying the number of units of intensity into the number of units of duration. Pleasure and pain, then, are quantities possessing two dimensions, just as superficies possesses the two dimensions of length and breadth.

In almost every case, however, the intensity of feeling will change from moment to moment. In-

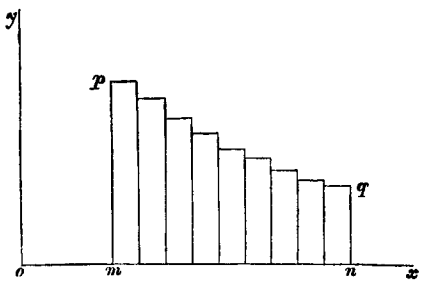

FIG. 1.

cessant variation characterises our states of mind, and this is the source of the main difficulties of the subject. Nevertheless, if these variations can be traced out at all, or any approach to method and law can be detected, it will be possible to form a conception of the resulting quantity of feeling. We may imagine that the intensity changes at the end of every minute, but remains constant in the intervals. The quantity during each minute may be represented, as in fig. 1, by a rectangle whose base is supposed to correspond to the duration of a minute, and whose

height is proportional to the intensity of the feeling during the minute in question. Along the line *ox* we measure *time*, and along parallels to the perpendicular line *oy* we measure *intensity*. Each of the rectangles between *pm* and *qn* represents the feeling of one minute. The aggregate quantity of feeling generated during the time *mn* will then be represented by the aggregate area of the rectangles

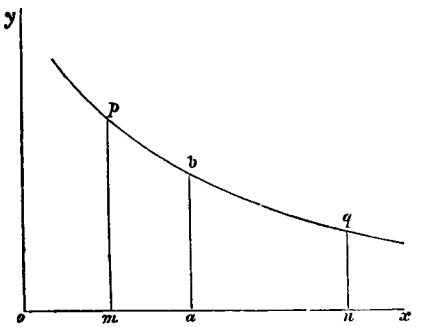

FIG. 2.

between *pm* and *qn*. In this case the intensity of the feeling is supposed to be gradually declining.

But it is an artificial assumption that the intensity would vary by sudden steps and at regular intervals. The error thus introduced will not be great if the intervals of time are very short, and will be less the shorter the intervals are made. To avoid all error, we must imagine the intervals of time to be infinitely short; that is, we must treat the intensity as varying continuously. Thus the proper representation of the variation of feeling is found in a curve of more or less complex character.

In fig. 2 the height of each point of the curve *pq*, above the horizontal line *ox*, indicates the intensity of feeling in a moment of time; and the whole quantity of feeling generated in the time *mn* is measured by the area bounded by the lines *pm*, *qn*, *mn*, and *pq*. The feeling belonging to any other time, *ma*, will be measured by the space *mabp* cut off by the perpendicular line *ab*.

Pain the Negative of Pleasure

It will be readily conceded that pain is the opposite of pleasure; so that to decrease pain is to increase pleasure; to add pain is to decrease pleasure. Thus we may treat pleasure and pain as positive and negative quantities are treated in algebra. The algebraic sum of a series of pleasures and pains will be obtained by adding the pleasures together and the pains together, and then striking the balance by subtracting the smaller amount from the greater. Our object will always be to maximise the resulting sum in the direction of pleasure, which we may fairly call the positive direction. This object we shall accomplish by accepting everything, and undertaking every action of which the resulting pleasure exceeds the pain which is undergone; we must avoid every object or action which leaves a balance in the other direction.

The most important parts of the theory will turn upon the exact equality, without regard to sign, of

the pleasure derived from the possession of an object, and the pain encountered in its acquisition. I am glad, therefore, to quote the following passage from Mr. Bain's treatise on *The Emotions and the Will*,[1] in which he exactly expresses the opposition of pleasure and pain:—"When pain is followed by pleasure, there is a tendency in the one, more or less, to neutralise the other. When the pleasure exactly assuages the pain, we say that the two are equivalent, or equal in amount, although of opposite nature, like hot and cold, positive and negative; and when two different kinds of pleasure have the power of satiating the same amount of pain, there is fair ground for pronouncing them of equal emotional power. Just as acids are pronounced equivalent when in amount sufficient to neutralise the same portion of alkali, and as heat is estimated by the quantity of snow melted by it, so pleasures are fairly compared as to their total efficacy on the mind, by the amount of pain that they are capable of submerging. In this sense there may be an effective estimate of degree."

Anticipated Feeling

Bentham has stated[2] that one of the main elements in estimating the force of a pleasure or pain is its *propinquity* or *remoteness*. It is certain that a very large part of what we experience in life depends not on the actual circumstances of the

[1] 1st ed., p. 30. [2] See above, p. 28.

moment so much as on the anticipation of future events. As Mr. Bain says,[1] "The foretaste of pleasure is pleasure begun: every actual delight casts before it a corresponding ideal." Every one must have felt that the enjoyment actually experienced at any moment is but limited in amount, and usually fails to answer to the anticipations which have been formed. "Man never is but always to be blest" is a correct description of our ordinary state of mind; and there is little doubt that, in minds of much intelligence and foresight, the greatest force of feeling and motive arises from the anticipation of a long-continued future.

Now, between the actual amount of feeling anticipated and that which is felt there must be some natural relation, very variable no doubt according to circumstances, the intellectual standing of the race, or the character of the individual; and yet subject to some general laws of variation. The intensity of present anticipated feeling must, to use a mathematical expression, be *some function of the future actual feeling and of the intervening time*, and it must increase as we approach the moment of realisation. The change, again, must be less rapid the farther we are from the moment, and more rapid as we come nearer to it. An event which is to happen a year hence affects us on the average about as much one day as another; but an event of importance, which is to take place three days hence, will probably

[1] *The Emotions and the Will*, 1st ed., p. 74.

affect us on each of the intervening days more acutely than the last.[1]

This power of anticipation must have a large influence in Economics; for upon it is based all accumulation of stocks of commodity to be consumed at a future time. That class or race of men who have the most foresight will work most for the future. The untutored savage, like the child, is wholly occupied with the pleasures and the troubles of the moment; the morrow is dimly felt; the limit of his horizon is but a few days off. The wants of a future year, or of a lifetime, are wholly unforeseen. But, in a state of civilisation, a vague though powerful feeling of the future is the main incentive to industry and saving. The cares of the moment are but ripples on the tide of achievement and hope. We may safely call that man happy who, however lowly his position and limited his possessions, can always hope for more than he has, and can feel that every moment of exertion tends to realise his aspirations. He, on the contrary, who seizes the enjoyment of the passing moment without regard to coming times, must discover sooner or later that his stock of pleasure is on the wane, and that even hope begins to fail.

Uncertainty of Future Events

In admitting the force of anticipated feeling, we are compelled to take account of the uncertainty of

[1] Meaning presumably: '. . . more acutely than on the previous day.'—[Ed.]

all future events. We ought never to estimate the value of that which may or may not happen as if it would certainly happen. When it is as likely as not that I shall receive £100, the chance is worth but £50, because if, for a great many times in succession, I purchase the chance at this rate, I shall almost certainly neither lose nor gain. The test of correct estimation of probabilities is that the calculations agree with fact on the average. If we apply this rule to all future interests, we must reduce our estimate of any feeling in the ratio of the numbers expressing the probability of its occurrence. If the probability is only one in ten that I shall have a certain day of pleasure, I ought to anticipate the pleasure with one-tenth of the force which would belong to it if certain. In selecting a course of action which depends on uncertain events, as, in fact, does everything in life, I should multiply the quantity of feeling attaching to every future event by the fraction denoting its probability. A great casualty, which is very unlikely to happen, may not be so important as a slight casualty which is nearly sure to happen. Almost unconsciously we make calculations of this kind more or less accurately in all the ordinary affairs of life; and in systems of life, fire, marine, or other insurance, we carry out the calculations to great perfection. In all industry directed to future purposes, we must take similar account of our want of knowledge of what is to be.

CHAPTER III

THEORY OF UTILITY

Definition of Terms

PLEASURE and pain are undoubtedly the ultimate objects of the Calculus of Economics. To satisfy our wants to the utmost with the least effort—to procure the greatest amount of what is desirable at the expense of the least that is undesirable—in other words, *to maximise pleasure*, is the problem of Economics. But it is convenient to transfer our attention as soon as possible to the physical objects or actions which are the source to us of pleasures and pains. A very large part of the labour of any community is spent upon the production of the ordinary necessaries and conveniences of life, such as food, clothing, buildings, utensils, furniture, ornaments, etc.; and the aggregate of these things, therefore, is the immediate object of our attention.

It is desirable to introduce at once, and to define, some terms which facilitate the expression of the Principles of Economics. By a *commodity* we shall

understand any object, substance, action, or service, which can afford pleasure or ward off pain. The name was originally abstract, and denoted the quality of anything by which it was capable of serving man. Having acquired, by a common process of confusion, a concrete signification, it will be well to retain the word entirely for that signification, and employ the term *utility* to denote the abstract quality whereby an object serves our purposes, and becomes entitled to rank as a commodity. Whatever can produce pleasure or prevent pain *may* possess utility. J.-B. Say has correctly and briefly defined utility as "la faculté qu'ont les choses de pouvoir servir à l'homme, de quelque manière que ce soit." The food which prevents the pangs of hunger, the clothes which fend off the cold of winter, possess incontestable utility; but we must beware of restricting the meaning of the word by any moral considerations. Anything which an individual is found to desire and to labour for must be assumed to possess for him utility. In the science of Economics we treat men not as they ought to be, but as they are. Bentham, in establishing the foundations of Moral Science in his great *Introduction to the Principles of Morals and Legislation* (page 3), thus comprehensively defines the term in question: "By utility is meant that property in any object, whereby it tends to produce benefit, advantage, pleasure, good, or happiness (all this, in the present case, comes to the same thing), or (what comes again to the same thing) to prevent the happening of mischief, pain, evil, or unhappiness to the party whose interest is considered."

This perfectly expresses the meaning of the word in Economics, provided that the will or inclination of the person immediately concerned is taken as the sole criterion, for the time, of what is or is not useful.

The Laws of Human Want

Economics must be founded upon a full and accurate investigation of the conditions of utility; and, to understand this element, we must necessarily examine the wants and desires of man. We, first of all, need a theory of the consumption of wealth. J. S. Mill, indeed, has given an opinion inconsistent with this. "Political economy," he says,[1] "has nothing to do with the consumption of wealth, further than as the consideration of it is inseparable from that of production, or from that of distribution. We know not of any laws of the consumption of wealth, as the subject of a distinct science; they can be no other than the laws of human enjoyment."

But it is surely obvious that Economics does rest upon the laws of human enjoyment; and that, if those laws are developed by no other science, they must be developed by economists. We labour to produce with the sole object of consuming, and the kinds and amounts of goods produced must be determined with regard to what we want to consume.

[1] *Essays on some Unsettled Questions of Political Economy*, p. 132.

Every manufacturer knows and feels how closely he must anticipate the tastes and needs of his customers: his whole success depends upon it; and, in like manner, the theory of Economics must begin with a correct theory of consumption. Many economists have had a clear perception of this truth. Lord Lauderdale distinctly states,[1] that "the great and important step towards ascertaining the causes of the direction which industry takes in nations . . . seems to be the discovery of what dictates the proportion of demand for the various articles which are produced." Senior, in his admirable treatise, has also recognised this truth, and pointed out what he calls the *Law of Variety* in human requirements. The necessaries of life are so few and simple, that a man is soon satisfied in regard to these, and desires to extend his range of enjoyment. His first object is to vary his food; but there soon arises the desire of variety and elegance in dress; and to this succeeds the desire to build, to ornament, and to furnish—tastes which, where they exist, are absolutely insatiable, and seem to increase with every improvement in civilisation.[2]

Many French economists also have observed that human wants are the ultimate subject-matter of Economics; Bastiat, for instance, in his *Harmonies*

[1] *Inquiry into the Nature and Origin of Public Wealth*, 2nd ed., 1819, p. 306 (1st ed. 1804).

[2] *Encyclopædia Metropolitana*, article "Political Economy," p. 133. 5th ed. of Reprint, p. 11.

of Political Economy, says,[1] "*Wants, Efforts, Satisfaction*—this is the circle of Political Economy."

In still later years, Courcelle-Seneuil actually commenced his treatise with a definition of *want*—"Le besoin économique est un désir qui a pour but la possession et la jouissance d'un objet matériel."[2] And I conceive that he has given the best possible statement of the problem of Economics when he expresses its object as "à satisfaire nos besoins avec la moindre somme de travail possible."[3]

Professor Hearn also begins his excellent treatise, entitled *Plutology, or the Theory of Efforts to supply Human Wants*, with a chapter in which he considers the nature of the wants impelling man to exertion.

The writer, however, who seems to me to have reached the deepest comprehension of the foundations of Economics is T. E. Banfield. His course of lectures delivered in the University of Cambridge in 1844, and published under the title of *The Organisation of Labour*, is highly interesting, though not always correct. In the following passage[4] he profoundly points out that the scientific basis of Economics is in a theory of consumption: I need make no excuse for quoting this passage at full length.

"The lower wants man experiences in common

[1] *Harmonies of Political Economy*, translated by P. J. Stirling, 1860, p. 65.

[2] *Traité Théorique et Pratique d'Economie Politique*, par J. G. Courcelle-Seneuil, 2me ed., Paris, 1867, tom. i. p. 25.

[3] *Ib.*, p. 33.

[4] 2nd ed., p. 11.

with brutes. The cravings of hunger and thirst, the effects of heat and cold, of drought and damp, he feels with more acuteness than the rest of the animal world. His sufferings are doubtless sharpened by the consciousness that he has no right to be subject to such inflictions. Experience, however, shows that privations of various kinds affect men differently in degree according to the circumstances in which they are placed. For some men the privation of certain enjoyments is intolerable, whose loss is not even felt by others. Some, again, sacrifice all that others hold dear for the gratification of longings and aspirations that are incomprehensible to their neighbours. Upon this complex foundation of low wants and high aspirations the Political Economist has to build the theory of production and consumption.

"An examination of the nature and intensity of man's wants shows that this connection between them gives to Political Economy its scientific basis. The first proposition of the theory of consumption is, that *the satisfaction of every lower want in the scale creates a desire of a higher character.* If the higher desire existed previous to the satisfaction of the primary want, it becomes more intense when the latter is removed. The removal of a primary want commonly awakens the sense of more than one secondary privation: thus a full supply of ordinary food not only excites to delicacy in eating, but awakens attention to clothing. The highest grade in the scale of wants, that of pleasure derived from the

beauties of nature and art, is usually confined to men who are exempted from all the lower privations. Thus the demand for, and the consumption of, objects of refined enjoyment has its lever in the facility with which the primary wants are satisfied. This, therefore, is the key to the true theory of value. Without relative value in the objects to the acquirement of which we direct our power, there would be no foundation for Political Economy as a science."

Utility is not an Intrinsic Quality

My principal work now lies in tracing out the exact nature and conditions of utility. It seems strange indeed that economists have not bestowed more minute attention on a subject which doubtless furnishes the true key to the problem of Economics.

In the first place, utility, though a quality of things, is *no inherent quality.* It is better described as *a circumstance of things* arising out of their relation to man's requirements. As Senior most accurately says, "Utility denotes no intrinsic quality in the things which we call useful; it merely expresses their relations to the pains and pleasures of mankind." We can never, therefore, say absolutely that some objects have utility and others have not. The ore lying in the mine, the diamond escaping the eye of the searcher, the wheat lying unreaped, the fruit ungathered for want of consumers, have no utility at all. The most wholesome and necessary kinds of

food are useless unless there are hands to collect and mouths to eat them sooner or later. Nor, when we consider the matter closely, can we say that all portions of the same commodity possess equal utility. Water, for instance, may be roughly described as the most useful of all substances. A quart of water per day has the high utility of saving a person from dying in a most distressing manner. Several gallons a day may possess much utility for such purposes as cooking and washing; but after an adequate supply is secured for these uses, any additional quantity is a matter of comparative indifference. All that we can say, then, is, that water, up to a certain quantity, is indispensable; that further quantities will have various degrees of utility; but that beyond a certain quantity the utility sinks gradually to zero; it may even become negative, that is to say, further supplies of the same substance may become inconvenient and hurtful.

Exactly the same considerations apply more or less clearly to every other article. A pound of bread per day supplied to a person saves him from starvation, and has the highest conceivable utility. A second pound per day has also no slight utility: it keeps him in a state of comparative plenty, though it be not altogether indispensable. A third pound would begin to be superfluous. It is clear, then, that *utility is not proportional to commodity*: the very same articles vary in utility according as we already possess more or less of the same article. The like may be said of other things. One suit of clothes per annum is

necessary, a second convenient, a third desirable, a fourth not unacceptable; but we, sooner or later, reach a point at which further supplies are not desired with any perceptible force, unless it be for subsequent use.

Law of the Variation of Utility

Let us now investigate this subject a little more closely. Utility must be considered as measured by, or even as actually identical with, the addition made to a person's happiness. It is a convenient name for the aggregate of the favourable balance of feeling produced—the sum of the pleasure created and the pain prevented. We must now carefully discriminate between the *total utility* arising from any commodity and the utility attaching to any particular portion of it. Thus the total utility of the food we eat consists in maintaining life, and may be considered as infinitely great; but if we were to subtract a tenth part from what we eat daily, our loss would be but slight. We should certainly not lose a tenth part of the whole utility of food to us. It might be doubtful whether we should suffer any harm at all.

Let us imagine the whole quantity of food which a person consumes on an average during twenty-four hours to be divided into ten equal parts. If his food be reduced by the last part, he will suffer but little; if a second tenth part be deficient, he will feel the want distinctly; the subtraction of the third tenth

part will be decidedly injurious ; with every subsequent subtraction of a tenth part his sufferings will be more and more serious, until at length he will be upon the verge of starvation. Now, if we call each of the tenth parts *an increment*, the meaning of these facts is, that each increment of food is less necessary, or possesses less utility, than the previous one. To explain this variation of utility we may make use of space-representations, which I have found convenient

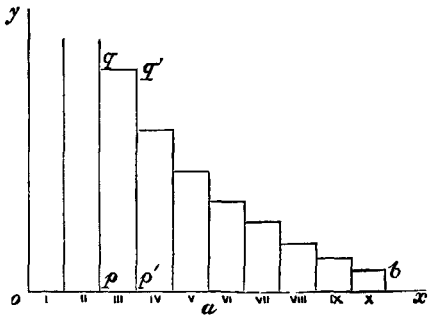

FIG. 3.

in illustrating the laws of Economics in my College lectures during fifteen years past.

Let the line *ox* be used as a measure of the quantity of food, and let it be divided into ten equal parts to correspond to the ten portions of food mentioned above. Upon these equal lines are constructed rectangles, and the area of each rectangle may be assumed to represent the utility of the increment of food corresponding to its base. Thus the utility of the last increment is small, being proportional to the small rectangle on x. As we approach towards *o*, each

increment bears a larger rectangle, that standing upon III being the largest complete rectangle. The utility of the next increment, II, is undefined, as also that of I, since these portions of food would be indispensable to life, and their utility, therefore, infinitely great.

We can now form a clear notion of the utility of the whole food, or of any part of it, for we have only to add together the proper rectangles. The utility of the first half of the food will be the sum of the rectangles standing on the line *oa* ; that of the second half will be represented by the sum of the smaller rectangles between *a* and *b*. The total utility of the food will be the whole sum of the rectangles, and will be infinitely great.

The comparative utility of the several portions is, however, the most important point. Utility may be treated [1] as *a quantity of two dimensions*, one dimension consisting in the quantity of the commodity, and another in the intensity of the effect produced upon the consumer. Now, the quantity of the commodity is measured on the horizontal line *ox*, and the intensity of utility will be measured by the length of the upright lines, or *ordinates*. The intensity of utility of the third increment is measured either by *pq*, or *p'q'*, and its utility is the product of the units in *pp'* multiplied by those in *pq*.

But the division of the food into ten equal parts is an arbitrary supposition. If we had taken twenty

[1] The theory of dimensions of utility is fully stated in a subsequent section.

or a hundred or more equal parts, the same general principle would hold true, namely, that each small portion would be less useful and necessary than the last. The law may be considered to hold true theoretically, however small the increments are made; and in this way we shall at last reach a figure which is undistinguishable from a continuous curve. The notion of infinitely small quantities of food may seem absurd as regards the consumption of one individual; but, when we consider the consumption of a nation as a whole, the consumption may well be conceived to increase or diminish by quantities which are, practically speaking, infinitely small compared with the whole consumption. The laws which we are about to trace out are to be conceived as theoretically true of the individual; they can only be practically verified as regards the aggregate transactions, productions, and consumptions of a large body of people. But the laws of the aggregate depend of course upon the laws applying to individual cases.

The law of the variation of the degree of utility of food may thus be represented by a continuous curve *pbq* (fig. 4), and the perpendicular height of each point of the curve above the line *ox*, represents the degree of utility of the commodity when a certain amount has been consumed.

Thus, when the quantity *oa* has been consumed, the degree of utility corresponds to the length of the line *ab*; for if we take a very little more food, *aa'*, its utility will be the product of *aa'* and *ab* very

nearly, and more nearly the less is the magnitude of *aa'*. The degree of utility is thus properly measured

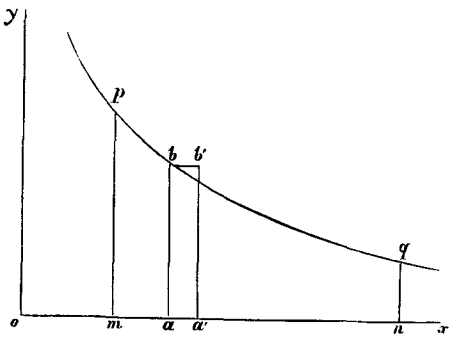

FIG. 4.

by the height of a very narrow rectangle corresponding to a very small quantity of food, which theoretically ought to be infinitely small.

Total Utility and Degree of Utility

We are now in a position to appreciate perfectly the difference between the *total utility* of any commodity and *the degree of utility* of the commodity at any point. These are, in fact, quantities of altogether different kinds, the first being represented by an area, and the second by a line. We must consider how we may express these notions in appropriate mathematical language.

Let x signify, as is usual in mathematical books, the quantity which varies independently—in this case the quantity of commodity. Let u denote the *whole utility* proceeding from the consumption of x.

Then u will be, as mathematicians say, *a function of* x; that is, it will vary in some continuous and regular, but probably unknown, manner, when x is made to vary. Our great object at present, however, is to express the *degree of utility.*

Mathematicians employ the sign Δ prefixed to a sign of quantity, such as x, to signify that a quantity of the same nature as x, but small in proportion to x, is taken into consideration. Thus Δx means a small portion of x, and $x+\Delta x$ is therefore a quantity a little greater than x. Now, when x is a quantity of commodity, the utility of $x+\Delta x$ will be more than that of x as a general rule. Let the whole utility of $x+\Delta x$ be denoted by $u+\Delta u$; then it is obvious that the increment of utility Δu belongs to the increment of commodity Δx; and if, for the sake of argument, we suppose the degree of utility uniform over the whole of Δx, which is nearly true owing to its smallness, we shall find the corresponding degree of utility by dividing Δu by Δx.

We find these considerations fully illustrated by fig. 4, in which oa represents x, and ab is the degree of utility at the point a. Now, if we increase x by the small quantity aa', or Δx, the utility is increased by the small rectangle $abb'a'$, or Δu; and, since a rectangle is the product of its sides, we find that the length of the line ab, the degree of utility, is represented by the fraction $\frac{\Delta u}{\Delta x}$.

As already explained, however, the utility of a

commodity may be considered to vary with perfect continuity, so that we commit a small error in assuming it to be uniform over the whole increment Δx. To avoid this we must imagine Δx to be reduced to an infinitely small size, Δu decreasing with it. The smaller the quantities are the more nearly we shall have a correct expression for ab, the degree of utility at the point a. Thus the *limit* of this fraction $\frac{\Delta u}{\Delta x}$, or, as it is commonly expressed, $\frac{du}{dx}$, is the degree of utility corresponding to the quantity of commodity x. *The degree of utility is*, in mathematical language, *the differential coefficient of u considered as a function of x*, and will itself be another function of x.

We shall seldom need to consider the degree of utility except as regards the last increment which has been consumed, or, which comes to the same thing, the next increment which is about to be consumed. I shall therefore commonly use the expression *final degree of utility*, as meaning the degree of utility of the last addition, or the next possible addition of a very small, or infinitely small, quantity to the existing stock. In ordinary circumstances, too, the final degree of utility will not be great compared with what it might be. Only in famine or other extreme circumstances do we approach the higher degrees of utility. Accordingly, we can often treat the lower portions of the curves of variation (*pbq*, fig. 4) which concern ordinary commercial transactions, while we

leave out of sight the portions beyond p or q. It is also evident that we may know the degree of utility at any point while ignorant of the total utility, that is, the area of the whole curve. To be able to estimate the total enjoyment of a person would be an interesting thing, but it would not be really so important as to be able to estimate the additions and subtractions to his enjoyment, which circumstances occasion. In the same way a very wealthy person may be quite unable to form any accurate statement of his aggregate wealth; but he may nevertheless have exact accounts of income and expenditure, that is, of additions and subtractions.

Variation of the Final Degree of Utility

The final degree of utility is that function upon which the Theory of Economics will be found to turn. Economists, generally speaking, have failed to discriminate between this function and the total utility, and from this confusion has arisen much perplexity. Many commodities which are most useful to us are esteemed and desired but little. We cannot live without water, and yet in ordinary circumstances we set no value on it. Why is this? Simply because we usually have so much of it that its final degree of utility is reduced nearly to zero. We enjoy, every day, the almost infinite utility of water, but then we do not need to consume more than we have. Let the supply run short by drought, and we begin

to feel the higher degrees of utility, of which we think but little at other times.

The variation of the function expressing the final degree of utility is the all-important point in economic problems. We may state as a general law, that *the degree of utility varies with the quantity of commodity, and ultimately decreases as that quantity increases.* No commodity can be named which we continue to desire with the same force, whatever be the quantity already in use or possession. All our appetites are capable of *satisfaction* or *satiety* sooner or later, in fact, both these words mean, etymologically, that we have had *enough*, so that more is of no use to us. It does not follow, indeed, that the degree of utility will always sink to zero. This may be the case with some things, especially the simple animal requirements, such as food, water, air, etc. But the more refined and intellectual our needs become, the less are they capable of satiety. To the desire for articles of taste, science, or curiosity, when once excited, there is hardly a limit.

This great principle of the ultimate decrease of the final degree of utility of any commodity is implied in the writings of many economists, though seldom distinctly stated. It is the real law which lies at the basis of Senior's so-called "Law of Variety." Indeed, Senior incidentally states the law itself. He says: "It is obvious that our desires do not aim so much at quantity as at diversity. Not only are there limits to the pleasure which commodities of any given

class can afford, but the pleasure diminishes in a rapidly increasing ratio long before those limits are reached. Two articles of the same kind will seldom afford twice the pleasure of one, and still less will ten give five times the pleasure of two. In proportion, therefore, as any article is abundant, the number of those who are provided with it, and do not wish, or wish but little, to increase their provision, is likely to be great; and, so far as they are concerned, the additional supply loses all, or nearly all, its utility. And, in proportion to its scarcity, the number of those who are in want of it, and the degree in which they want it, are likely to be increased; and its utility, or, in other words, the pleasure which the possession of a given quantity of it will afford, increases proportionally." [1]

Banfield's "Law of the Subordination of Wants" also rests upon the same basis. It cannot be said, with accuracy, that the satisfaction of a lower want *creates* a higher want; it merely permits the higher want to manifest itself. We distribute our labour and possessions in such a way as to satisfy the more pressing wants first. If food runs short, the all-absorbing question is, how to obtain more, because, at the moment, more pleasure or pain depends upon food than upon any other commodity. But, when food is moderately abundant, its final degree of utility falls very low, and wants of a more complex and less satiable nature become comparatively prominent.

[1] *Encyclopædia Metropolitana*, p. 133. Reprint, p. 12.

The writer, however, who appears to me to have most clearly appreciated the nature and importance of the law of utility, is Richard Jennings, who, in 1855, published a small book called the *Natural Elements of Political Economy*.[1] This work treats of the physical groundwork of Economics, showing its dependence on physiological laws. It displays great insight into the real basis of Economics; yet I am not aware that economists have bestowed the slightest attention on Jennings's views.[2] I give, therefore, a full extract from his remarks on the nature of utility. It will be seen that the law, as I state it, is no novelty, and that careful deduction from principles in our possession is alone needed to give us a correct Theory of Economics.

"To turn from the relative effect of commodities, in producing sensations, to those which are absolute, or dependent only on the quantity of each commodity, it is but too well known to every condition of men, that the degree of each sensation which is produced, is by no means commensurate with the quantity of the commodity applied to the senses. . . . These effects require to be closely observed, because they are the foundation of the changes of money price, which valuable objects command in times of varied scarcity and abundance; we shall therefore

[1] London: Longmans.

[2] Cairnes is, however, an exception. See his work on *The Character and Logical Method of Political Economy*. London, 1857, p. 81. 2nd ed. (Macmillan), 1875, pp. 56, 110, 224 App. B.

here direct our attention to them for the purpose of ascertaining the nature of the law according to which the sensations that attend on consumption vary in degree with changes in the quantity of the commodity consumed.

"We may gaze upon an object until we can no longer discern it, listen until we can no longer hear, smell until the sense of odour is exhausted, taste until the object becomes nauseous, and touch until it becomes painful; we may consume food until we are fully satisfied, and use stimulants until more would cause pain. On the other hand, the same object offered to the special senses for a moderate duration of time, and the same food or stimulants consumed when we are exhausted or weary, may convey much gratification. If the whole quantity of the commodity consumed during the interval of these two states of sensation, the state of satiety and the state of inanition, be conceived to be divided into a number of equal parts, each marked with its proper degrees of sensation, the question to be determined will be, what relation does the difference in the degrees of the sensation bear to the difference in the quantities of the commodity?

"First, with respect to all commodities, our feelings show that the degrees of satisfaction do not proceed *pari passu* with the quantities consumed; they do not advance equally with each instalment of the commodity offered to the senses, and then suddenly stop; but diminish gradually, until they

ultimately disappear, and further instalments can produce no further satisfaction. In this progressive scale the increments of sensation resulting from equal increments of the commodity are obviously less and less at each step,—each degree of sensation is less than the preceding degree. Placing ourselves at that middle point of sensation, the *juste milieu*, the *aurea mediocritas*, the ἀριστον μετρον of sages, which is the most usual status of the mass of mankind, and which, therefore, is the best position that can be chosen for measuring deviations from the usual amount, we may say that the law which expresses the relation of degrees of sensation to quantities of commodities is of this character: if the average or temperate quantity of commodities be increased, the satisfaction derived is increased in a less degree, and ultimately ceases to be increased at all; if the average or temperate quantity be diminished, the loss of more and more satisfaction will continually ensue, and the detriment thence arising will ultimately become exceedingly great."[1]

Disutility and Discommodity

A few words will suffice to suggest that as utility corresponds to the production of pleasure, or, at least, a favourable alteration in the balance of pleasure and pain, so negative utility will consist in the production of pain, or the unfavourable alteration of the balance. In reality we must be almost as often

[1] Pp. 96-9.

concerned with the one as with the other; nevertheless, economists have not employed any distinct technical terms to express that production of pain, which accompanies so many actions of life. They have fixed their attention on the more agreeable aspect of the matter. It will be allowable, however, to appropriate the good English word *discommodity*, to signify any substance or action which is the opposite of *commodity*, that is to say, *anything which we desire to get rid of*, like ashes or sewage. Discommodity is, indeed, properly an abstract form signifying inconvenience, or disadvantage; but, as the noun *commodities* has been used in the English language for four hundred years at least as a concrete term,[1] so we may now convert discommodity into a concrete term, and speak of *discommodities* as substances or things which possess the quality of causing inconvenience or harm. For the abstract notion, the opposite or negative of utility, we may invent the term *disutility*, which will mean something different from inutility, or the absence of utility. It is obvious that utility passes through inutility before changing into disutility, these notions being related as +, 0 and —.

Distribution of Commodity in different Uses

The principles of utility may be illustrated by

[1] It is used precisely in its present economic sense in the remarkable "Processe of the Libelle of English Policie," probably written in the fifteenth century, and printed in Hakluyt's *Voyages*.

considering the mode in which we distribute a commodity when it is capable of several uses. There are articles which may be employed for many distinct purposes: thus, barley may be used either to make beer, spirits, bread, or to feed cattle; sugar may be used to eat, or for producing alcohol; timber may be used in construction, or as fuel; iron and other metals may be applied to many different purposes. Imagine, then, a community in the possession of a certain stock of barley; what principles will regulate their mode of consuming it? Or, as we have not yet reached the subject of exchange, imagine an isolated family, or even an individual, possessing an adequate stock, and using some in one way and some in another. The theory of utility gives, theoretically speaking, a complete solution of the question.

Let s be the whole stock of some commodity, and let it be capable of two distinct uses. Then we may represent the two quantities appropriated to these uses by x_1 and y_1, it being a condition that $x_1 + y_1 = s$. The person may be conceived as successively expending small quantities of the commodity. Now it is the inevitable tendency of human nature to choose that course which appears to offer the greatest advantage at the moment. Hence, when the person remains satisfied with the distribution he has made, it follows that no alteration would yield him more pleasure; which amounts to saying that an increment of commodity would yield exactly as much utility in one use as in another. Let Δu_1, Δu_2, be the incre-

ments of utility, which might arise respectively from consuming an increment of commodity in the two different ways. When the distribution is completed, we ought to have $\Delta u_1 = \Delta u_2$; or at the limit we have the equation

$$\frac{du_1}{dx} = \frac{du_2}{dy},$$

which is true when x, y are respectively equal to x_1, y_1. We must, in other words, have the *final degrees of utility* in the two uses equal.

The same reasoning which applies to uses of the same commodity will evidently apply to any two uses, and hence to all uses simultaneously, so that we obtain a series of equations less numerous by a unit than the number of ways of using the commodity. The general result is that commodity, if consumed by a perfectly wise being, must be consumed with a maximum production of utility.

We should often find these equations to fail. Even when x is equal to $\frac{99}{100}$ of the stock, its degree of utility might still exceed the utility attaching to the remaining $\frac{1}{100}$ part in either of the other uses. This would mean that it was preferable to give the whole commodity to the first use. Such a case might perhaps be said to be not the exception but the rule; for, whenever a commodity is capable of only one use, the circumstance is theoretically represented by saying, that the final degree of utility in this employment always exceeds that in any other employment.

Under peculiar circumstances great changes may take place in the consumption of a commodity. In a time of scarcity the utility of barley as food might rise so high as to exceed altogether its utility, even as regards the smallest quantity, in producing alcoholic liquors; its consumption in the latter way would then cease. In a besieged town the employment of articles becomes revolutionised. Things of great utility in other respects are ruthlessly applied to strange purposes. In Paris a vast stock of horses were eaten, not so much because they were useless in other ways, as because they were needed more strongly as food. A certain stock of horses had, indeed, to be retained as a necessary aid to locomotion, so that the equation of the degrees of utility never wholly failed.

Theory of Dimensions of Economic Quantities

In the recent progress of physical science, it has been found requisite to use notation for the purpose of displaying clearly the natures and relations of its various kinds of quantities concerned. Each different sort of quantity is, of course, expressed in terms of its own appropriate unit—length in terms of yards, or metres; surface, or area, in terms of square yards or square metres; time in terms of seconds, days, or years; and so forth. But the more complicated quantities are evidently related to the simpler ones. Surface is measured by the *square yard*—that is to say, the unit of length is involved twice over, and if

by L we denote one dimension of length, then the dimensions of surface are LL, or L^2. The dimensions of cubic capacity are in like manner LLL, or L^3.

In these cases the dimensions all enter *positively*, because the number of units in the cubical body, for instance, is found by *multiplying* the numbers of units in its length, breadth, and depth. In other cases a dimension enters *negatively*. Thus denoting time by T, it is easy to see that the dimensions of velocity will be L *divided by* T, or LT^{-1}, because the number of units in the velocity of a body is found by *dividing* the units of length passed over by the units of time occupied in passing. In expressing the dimensions of thermal and electric quantities, fractional exponents often become necessary, and the subject assumes the form of a theory of considerable complexity. The reader to whom this branch of science is new will find a section briefly describing it in my *Principles of Science*, 3rd ed., p. 325, or he may refer to the works there mentioned.[1]

Now, if such a theory of dimensions is requisite in dealing with the precise ideas of physical magnitudes, it seems to be still more desirable as regards the quantities with which we are concerned in Economics. One of the first and most difficult steps in a science is to conceive clearly the nature of the magnitudes

[1] J. D. Everett's *Illustrations of the Centimetre-gramme-second System of Units*, 1875 [5th ed., 1902]; Fleeming Jenkin's *Text-Book of Electricity and Magnetism*, 1873; Clerk-Maxwell's *Theory of Heat*, or the commencement of his great *Treatise on Electricity*, vol. i. p. 2.

about which we are arguing. Heat was long the subject of discussion and experiment before physicists formed any definite idea how its quantity could be measured and connected with other physical quantities. Yet, until that was done, it could not be considered the subject of an exact science. For one or two centuries economists have been wrangling about wealth, demand and supply, value, production, capital, interest, and the like; but hardly any one could say exactly what were the natures of the quantities in question. Believing that it is in forming these primary ideas that we require to exercise the greatest care, I have thought it well worth the trouble and space to enter fully into a discussion of the dimensions of economic quantities.

Beginning with the easiest and simplest ideas, the *dimensions of commodity*, regarded merely as a physical quantity, will be *the dimensions of mass*. It is true that commodities are measured in various ways,—thread by length, carpet by length, corn and liquids by cubic measure, eggs by number, metals and most other goods by weight. But it is obvious that, though the carpet be sold by length, the breadth and the weight of the cloth are equally taken into account in fixing the terms of sale. There will generally be a tacit reference to weight, and through weight to mass of materials in all measurement of commodity. Even if this be not always the case, we may, for the sake of simplifying our symbols in the first treatment of the subject, assume that it is so. We need hardly recede

to any ultimate analysis of the physical conditions of the commodity, but may take it to be measured by *mass*, symbolised by M, the sign usually employed in physical science to denote this dimension.

A little consideration will show, however, that we have really little to do with absolute quantities of commodity. One hundred sacks of corn regarded merely by themselves can have no important meaning for the economist. Whether the quantity is large or small, enough or too much, depends in the first place upon the number of consumers for whom it is intended, and, in the second place, upon the time for which it is to last them. We may perhaps throw out of view the number of consumers in this theory, by supposing that we are always dealing with the single average individual, the unit of which population is made up. Still, we cannot similarly get rid of the element of time. Quantity of supply must necessarily be estimated by the number of units of commodity divided by the number of units in the time over which it is to be expended. Thus it will involve M positively and T negatively, and its dimensions will be presented by MT^{-1}. Thus in reality *supply should be taken to mean not supply absolutely, but rate of supply.*

Consumption of commodity must have the same dimensions. For goods must be consumed in time; any action or effect endures a greater or less time, and commodity which will be abundant for a less time may be scanty for a greater time. To say that

a town consumes fifty million gallons of water is unmeaning *per se.* Before we can form any judgment about the statement, we must know whether it is consumed in a day, or a week, or a month.

Following out this course of thought we shall arrive at the conclusion that time enters into all economic questions. We live in time, and think and act in time; we are in fact altogether the creatures of time. Accordingly it is rate of supply, rate of production, rate of consumption, per unit of time that we shall be really treating; but it does not follow that T^{-1} enters into all the dimensions with which we deal.

As was fully explained in Chapter II., the ultimate quantities which we treat in Economics are Pleasures and Pains, and our most difficult task will be to express their dimensions correctly. In the first place, pleasure and pain must be regarded as measured upon the same scale, and as having, therefore, the same dimensions, being quantities of the same kind, which can be added and subtracted; they differ only in sign or direction. Now, the only dimension belonging properly to feeling seems to be *intensity*, and this intensity must be independent both of time and of the quantity of commodity enjoyed. *The intensity of feeling must mean, then, the instantaneous state produced by an elementary or infinitesimal quantity of commodity consumed.*

Intensity of feeling, however, is only another name for degree of utility, which represents the

favourable effect produced upon the human frame by the consumption of commodity, that is by an elementary or infinitesimal quantity of commodity. Putting U to indicate this dimension, we must remember that U will not represent even the full dimensions of the instantaneous state of pleasure or pain, much less the continued state which extends over a certain duration of time. The instantaneous state depends upon the sufficiency or insufficiency of supply of commodity. To enjoy a highly pleasurable condition, a person must want a good deal of commodity, and must be well supplied with it. Now, this supply is, as already explained, rate of supply, so that we must multiply U by MT^{-1} in order to arrive at the real instantaneous state of feeling. The kind of quantity thus symbolised by MUT^{-1} must be interpreted as meaning *so much commodity producing a certain amount of pleasurable effect per unit of time.* But this quantity will not be *quantity of utility* itself. It will only be that quantity which, when multiplied by time, will produce quantity of utility. Pleasure, as was stated at the outset, has the dimensions of intensity and duration. It is then this *intensity* which is symbolised by MUT^{-1}, and we must multiply this last symbol by T in order to obtain the dimensions of utility or quantity of pleasure produced. But in making this multiplication, $MUT^{-1}T$ reduces to MU, which must therefore be taken to denote the dimensions of *quantity of utility.*

We here meet with an explanation of the fact, so

long perplexing to me, that the element of time does not appear throughout the diagrams and problems of this theory relating to utility and exchange. All goes on in time, and time is a necessary element of the question; yet it does not explicitly appear. Recurring to our diagrams, that for instance on p. 46, it is obvious that the dimension U, or degree of utility, is measured upon the perpendicular axis oy. The horizontal axis must, therefore, be that upon which rate of supply of commodity or MT^{-1} is measured, strictly speaking. If now we introduce the duration of the utility, we should apparently need a third axis, perpendicular to the plane of the page, upon which to denote it. But were we to introduce this third dimension, we should obtain a solid figure, representing a quantity truly of three dimensions. This would be erroneous, because the third dimension T enters negatively into the quantity represented by the horizontal axis. Thus time eliminates itself, and we arrive at a quantity of two dimensions correctly represented by a curvilinear area, one dimension of which corresponds to each of the factors in MU.

This result is at first sight paradoxical; but the difficulty is exactly analogous to that which occurs in the question of interest, and which led so profound a mathematician as Dean Peacock into a blunder, as will be shown in the Chapter upon Capital. Interest of money is proportional to the length of time for which the principal is lent, and also to the amount of money lent and the rate of interest. But this rate of

interest involves time negatively, so that time is ultimately eliminated, and interest emerges with the same dimensions as the principal sum. In the case of utility we begin with a certain absolute stock of commodity, M. In expending it we must spread it over more or less time, so that it is really rate of supply which is to be considered; but it is this rate MT^{-1}, not simply M, which influences the final degree of utility, U, at which it is consumed. If the same commodity be made to last a longer time, the degree of utility will be higher, because the necessity of the consumer will be less satisfied. Thus the absolute amount of utility produced will, as a general rule, be greater as the time of expenditure is greater; but this will also be the case with the quantity symbolised by MU, because the quantity U will under those circumstances be greater, while M remains constant.

To clear up the matter still further if possible, I will recapitulate the results we have arrived at.

M means absolute amount of commodity.

MT^{-1} means amount of commodity applied, so much per unit of time.

U means the resulting pleasurable effect of any increment of that supply, an infinitesimal quantity supplied per unit of time.

MUT^{-1} means therefore so much pleasurable effect produced per unit of commodity per unit of time.[1]

[1] Reference to the lines in italics on p. 66 shows that this should read: "MUT^{-1} means therefore so much pleasurable effect produced by *so much* commodity per unit of time."—[Ed.]

$MUT^{-1}T$, or MU, means therefore so much absolute pleasurable effect produced by commodity in an unspecified duration of time.

Actual, Prospective, and Potential Utility

The difficulties of Economics are mainly the difficulties of conceiving clearly and fully the conditions of utility. Even at the risk of being tiresome, I will therefore point out more minutely how various are the senses in which a thing may be said to have utility.

It is quite usual, and perhaps correct, to call iron or water or timber a useful substance; but we may mean by these words at least three distinct facts. We may mean that a particular piece of iron is at the present moment actually useful to some person; or that, although not actually useful, it is expected to be useful at a future time; or we may only mean that it would be useful if it were in the possession of some person needing it. The iron rails of a railway, the iron which composes the Britannia Bridge,[1] or an ocean steamer, is actually useful; the iron lying in a merchant's store is not useful at present, though it is expected soon to be so; but there is a vast quantity of iron existing in the bowels of the earth, which has all the physical properties of iron, and might be useful

[1] Over the Menai Straits, North Wales. This tubular bridge was considered a feat of engineering at the time of its completion in 1850, and for many years after.—[Ed.]

if extracted, though it never will be. These are instances of *actual, prospective, and potential utility.*

It will be apparent that *potential utility* does not really enter into the science of Economics, and when I speak of *utility* simply, I do not mean to include potential utility. It is a question of physical science whether a substance possesses qualities which might make it suitable to our needs if it were within our reach. Only when there arises some degree of probability, however slight, that a particular object will be needed, does it acquire *prospective utility*, capable of rendering it a desirable possession. As Condillac correctly remarks :[1] "On diroit que les choses ne commencent à exister pour eux, qu'au moment où ils ont un intérêt à savoir qu'elles existent." But a very large part in industry, and the science of industry, belongs to *prospective utility*. We can at any one moment use only a very small fraction of what we possess. By far the greater part of what we hold might be allowed to perish at any moment, without harm, if we could have it re-created with equal ease at a future moment, when need of it arises.

We might also distinguish, as is customary with French economists, between *direct* and *indirect utility*. Direct utility attaches to a thing like food, which we can actually supply to satisfy our wants. But things which have no direct utility may be the means of procuring us such by exchange, and they may therefore be said to have indirect utility.[1] To the latter form of utility I have elsewhere applied the name *acquired utility*.[2] This distinction is not the same as that which is made in the Theory of Capital between *mediate* and *immediate utility*, the former being that of any implement, machine, or other means of procuring commodities possessing *immediate* and *direct utility*—that is, the power of satisfying want.[3]

[1] Condillac, *Le Commerce et le Gouvernement*, Seconde Partie, Introduction. *Œuvres Complètes.* Paris, 1803. Tom. vii. p. 2.

Distribution of a Commodity in Time

We have seen that, when a commodity is capable of being used for different purposes, definite principles regulate its application to those purposes. A similar question arises when a stock of commodity is in hand, and must be expended over a certain interval of time more or less definite. The science of Economics must point out the mode of consuming it to the greatest advantage—that is, with a maximum result of utility. If we reckon all future pleasures and pains as if they were present, the solution will be the same as in the case of different uses. If a commodity has to be distributed over n days' use, and v_1, v_2, etc., be the final degrees of utility on each day's consumption, then we ought clearly to have

$$v_1 = v_2 = v_3 = \ldots = v_n.$$

[1] Garnier, *Traité d'Economie Politique*, 5me ed., p. 11.

[2] See chap. iv., p. 137.

[3] See chap. vii. [These terms are not used in chap. vii., or elsewhere in this book.—Ed.]

It may, however, be uncertain during how many days we may require the stock to last. The commodity might be of a perishable nature, so that if we were to keep some of it for ten days, it might become unserviceable, and its utility be sacrificed. Assuming that we can estimate more or less exactly the probability of its remaining good, let $p_1, p_2, p_3 \ldots p_{10}$, be these probabilities. Then, on the principle (p. 36) that a future pleasure or pain must be reduced in proportion to its want of certainty, we have the equations

$$v_1 p_1 = v_2 p_2 = \ldots = v_{10} p_{10}.$$

The general result is, that as the probability is less, the commodity assigned to each day is less, so that v, its final degree of utility, will be greater.

So far we have taken no account of the varying influence of an event according to its propinquity or remoteness. The distribution of commodity described is that which should be made, and would be made by a being of perfect good sense and foresight. To secure a maximum of benefit in life, all future events, all future pleasures or pains, should act upon us with the same force as if they were present, allowance being made for their uncertainty. The factor expressing the effect of remoteness should, in short, always be unity, so that time should have no influence. But no human mind is constituted in this perfect way: a future feeling is always less influential than a present one. To take this fact into account, let q_1, q_2, q_3, etc., be the undetermined fractions which express the

ratios of the present pleasures or pains to those future ones from whose anticipation they arise. Having a stock of commodity in hand, our tendency will be to distribute it so that the following equations will hold true—

$$v_1 p_1 q_1 = v_2 p_2 q_2 = v_3 p_3 q_3 = \ldots = v_n p_n q_n.$$

It will be an obvious consequence of these equations that less commodity will be assigned to future days in some proportion to the intervening time.

An illustrative problem, involving questions of prospective utility and probability, is found in the case of a vessel at sea, which is insufficiently victualled for the probable length of the voyage to the nearest port. The actual length of the voyage depends on the winds, and must be uncertain; but we may suppose that it will almost certainly last ten days or more, but not more than thirty days. It is apparent that if the food were divided into thirty equal parts, partial famine and suffering would be certainly endured for the first ten days, to ward off later evils which may not be encountered. To consume one-tenth part of the food on each of the first ten days would be still worse, as almost certainly entailing starvation on the following days. To determine the most beneficial distribution of the food, we should require to know the probability of each day between the tenth and thirtieth days forming part of the voyage, and also the law of variation of the degree of utility of food. The whole stock ought then to be divided into

thirty portions, allotted to each of the thirty days, and of such magnitudes that the final degrees of utility multiplied by the probabilities may be equal. Thus, let v_1, v_2, v_3, etc., be the final degrees of utility of the first, second, third, and other days supplied, and p_1, p_2, p_3, etc., the probabilities that the days in question will form part of the voyage; then we ought to have

$$p_1 v_1 = p_2 v_2 = p_3 v_3 = \ldots = p_{29} v_{29} = p_{30} v_{30}.$$

If these equations did not hold true, it would be beneficial to transfer a small portion from one lot to some other lot. As the voyage is supposed certainly to last the first ten days, we have

$$p_1 = p_2 = \ldots = p_{10} = 1;$$

hence we must have

$$v_1 = v_2 = \ldots = v_{10};$$

that is to say, the allotments to the first ten days should be equal. They should afterwards decrease according to some regular law; for, as the probability decreases, the final degree of utility should increase in inverse proportion.

CHAPTER IV

THEORY OF EXCHANGE

Importance of Exchange in Economics

EXCHANGE is so important a process in the maximising of utility and the saving of labour, that some economists have regarded their science as treating of this operation alone. Utility arises from commodities being brought in suitable quantities and at the proper times into the possession of persons needing them; and it is by exchange, more than any other means, that this is effected. Trade is not indeed the only method of economising: a single individual may gain in utility by a proper consumption of the stock in his possession. The best employment of labour and capital by a single person is also a question disconnected from that of exchange, and which must yet be treated in the science. But, with these exceptions, I am perfectly willing to agree with the high importance attributed to exchange.

It is impossible to have a correct idea of the science of Economics without a perfect comprehension

of the Theory of Exchange; and I find it both possible and desirable to consider this subject before introducing any notions concerning labour or the production of commodities. In these words of J. S. Mill I thoroughly concur: "Almost every speculation respecting the economical interests of a society thus constituted, implies some theory of Value: the smallest error on that subject infects with corresponding error all our other conclusions; and anything vague or misty in our conception of it creates confusion and uncertainty in everything else." But when he proceeds to say, "Happily, there is nothing in the laws of Value which remains for the present or any future writer to clear up; the theory of the subject is complete"[1]—he utters that which it would be rash to say of any of the sciences.

Ambiguity of the term Value

I must, in the first place, point out the thoroughly ambiguous and unscientific character of the term *value*. Adam Smith noticed the extreme difference of meaning between *value in use* and *value in exchange*; and it is usual for writers on Economics to caution their readers against the confusion of thought to which they are liable. But I do not believe that either writers or readers can avoid the confusion so long as they use the word. In spite of the most acute feeling of the danger, I often detect myself

[1] *Principles of Political Economy*, book iii., chap. i. sec. 1.

using the word improperly; nor do I think that the best authors escape the danger.

Let us turn to Mill's definition of Exchange Value,[1] and we see at once the misleading power of the term. He tells us—"Value is a relative term. The value of a thing means the quantity of some other thing, or of things in general, which it exchanges for." Now, if there is any fact certain about exchange value, it is, that it means not an object at all, but a circumstance of an object. Value implies, in fact, a relation; but if so, it cannot possibly be *some other thing*. A student of Economics has no hope of ever being clear and correct in his ideas of the science if he thinks of value as at all a *thing* or an *object*, or even as anything which lies in a thing or object. Persons are thus led to speak of such a nonentity as *intrinsic value*. There are, doubtless, qualities inherent in such a substance as gold or iron which influence its value; but the word Value, so far as it can be correctly used, merely expresses *the circumstance of its exchanging in a certain ratio for some other substance*.

Value expresses Ratio of Exchange

If a ton of pig-iron exchanges in a market for an ounce of standard gold, neither the iron is value nor the gold; nor is there value in the iron nor in the gold. The notion of value is concerned only in the

[1] *Principles of Political Economy*, book iii., chap. vi.

fact or circumstance of one exchanging for the other. Thus it is scientifically incorrect to say that the value of the ton of iron *is* the ounce of gold: we thus convert value into a concrete thing; and it is, of course, equally incorrect to say that the value of the ounce of gold is the ton of iron. The more correct and safe expression is, that *the value of the ton of iron is equal to the value of the ounce of gold*, or that their values are as one to one.

Value in exchange expresses nothing but a ratio, and the term should not be used in any other sense. To speak simply of the value of an ounce of gold is as absurd as to speak of the *ratio of the number seventeen*. What is the ratio of the number seventeen? The question admits no answer, for there must be another number named in order to make a ratio; and the ratio will differ according to the number suggested. What is the value of iron compared with that of gold? —is an intelligible question. The answer consists in stating the ratio of the quantities exchanged.

Popular use of the term Value

In the popular use of the word value no less than three distinct though connected meanings seem to be confused together. These may be described as

(1) Value in use;
(2) Esteem, or urgency of desire;
(3) Ratio of exchange.

Adam Smith, in the familiar passage already referred

to, distinguished between the first and the third meanings. He said,[1] "The word value, it is to be observed, has two different meanings, and sometimes expresses the power of purchasing other goods which the possession of that object conveys. The one may be called 'value in use'; the other 'value in exchange.' The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce anything; scarce anything can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it."

It is sufficiently plain that, when Smith speaks of water as being highly useful and yet devoid of purchasing power, he means *water in abundance*, that is to say, water so abundantly supplied that it has exerted its full useful effect, or its *total utility*. Water, when it becomes very scarce, as in a dry desert, acquires exceedingly great purchasing power. Thus Smith evidently means by value in use, *the total utility of a substance of which the degree of utility has sunk very low, because the want of such substance has been well nigh satisfied*. By purchasing power he clearly means the ratio of exchange for other commodities. But here he fails to point out that the

[1] *Wealth of Nations*, book i., chap. iv., near the end.

quantity of goods received in exchange depends just as much upon the nature of the goods received, as on the nature of those given for them. In exchange for a diamond we can get a great quantity of iron, or corn, or paving-stones, or other commodity of which there is abundance; but we can get very few rubies, sapphires, or other precious stones. Silver is of high purchasing power compared with zinc, or lead, or iron, but of small purchasing power compared with gold, platinum, or iridium. Yet we might well say in any case that diamond and silver are things of high value. Thus I am led to think that the word value is often used in reality to mean *intensity of desire or esteem for a thing*. A silver ornament is a beautiful object apart from all ideas of traffic; it may thus be valued or esteemed simply because it suits the taste and fancy of its owner, and is the only one possessed. Even Robinson Crusoe must have looked upon each of his possessions with varying esteem and desire for more, although he was incapable of exchanging with any other person. Now, in this sense value seems to be identical with the final degree of utility of a commodity, as defined in a previous page (p. 51); it is measured by the intensity of the pleasure or benefit which would be obtained from a new increment of the same commodity. No doubt there is a close connection between value in this meaning, and value as ratio of exchange. Nothing can have a high purchasing power unless it be highly esteemed in itself; but it may be highly esteemed apart from all comparison with other things; and, though highly esteemed, it may have a low purchasing power, because those things against which it is measured are still more esteemed.

Thus I come to the conclusion that, in the use of the word value, three distinct meanings are habitually confused together, and require to be thus distinguished—

(1) Value in use = total utility;
(2) Esteem = final degree of utility;
(3) Purchasing power = ratio of exchange.

It is not to be expected that we could profitably discuss such matters as economic doctrines, while the fundamental ideas of the subject are thus jumbled up together in one ambiguous word. The only thorough remedy consists in substituting for the dangerous name *value* that one of the three stated meanings which is intended in each case. In this work, therefore, I shall discontinue the use of the word value altogether, and when, as will be most often the case in the remainder of the book, I need to refer to the third meaning, often called by economists *exchange or exchangeable value*, I shall substitute the wholly unequivocal expression *Ratio of Exchange*, specifying at the same time what are the *two articles* exchanged. When we speak of the ratio of exchange of pig-iron and gold, there can be no possible doubt that we intend to refer to the ratio of the number of units of the one commodity to the

number of units of the other commodity for which it exchanges, the units being arbitrary concrete magnitudes, but the ratio an abstract number.

When I proposed, in the first edition of this book, to use Ratio of Exchange instead of the word value, the expression had been so little, if at all, employed by English economists, that it amounted to an innovation. J. S. Mill, indeed, in his chapters on Value, speaks once and again of things exchanging for each other "in the ratio of their cost of production"; but he always omits to say distinctly that exchange value is itself a matter of ratio. As to Ricardo, Malthus, Adam Smith, and other great English economists, although they usually discourse at some length upon the meanings of the word value, I am not aware that they ever explicitly apply the name *ratio* to exchange or exchangeable value. Yet ratio is unquestionably the correct scientific term, and the only term which is strictly and entirely correct.

It is interesting, therefore, to find that, although overlooked by English economists, the expression had been used by two or more of the truly scientific French economists, namely, Le Trosne and Condillac. Le Trosne carefully defines value in the following terms:[1] "La valeur consiste dans le rapport d'échange qui se trouve entre telle chose et telle autre, entre telle mesure d'une production et telle mesure des autres." Condillac apparently adopts the words of

[1] *De l'Intérêt Social*, 1777, chap. i. sec. 4.

Le Trosne, saying[1] of value: "Qu'elle consiste dans le rapport d'échange entre telle chose et telle autre." Such economical works as those of Baudeau, Le Trosne, and Condillac were almost wholly unknown to English readers until attention was drawn to them by Mr. H. D. Macleod and Professor Adamson; but I shall endeavour for the future to make proper use of them.

Dimension of Value

There is no difficulty in seeing that, when we use the word Value in the sense of ratio of exchange, its dimension will be simply zero. Value will be expressed, like angular magnitude and other ratios in general, by abstract number. Angular magnitude is measured by the ratio of a line to a line, the ratio of the arc subtended by the angle to the radius of the circle. So value in this sense is a ratio of the quantity of one commodity to the quantity of some other commodity exchanged for it. If we compare the commodities simply as physical quantities, we have the dimensions M divided by M, or MM^{-1}, or M^0. Exactly the same result would be obtained if, instead of taking the mere physical quantities, we were to compare their utilities, for we should then have MU divided by MU or M^0U^0, which, as it really means *unity*,[2] is identical in meaning with M^0.

[1] *Le Commerce et le Gouvernement*, 1776; *Œuvres Complètes de Condillac*, 1803, tom. 6me, p. 20.

[2] Correctly *no dimensions*; that is, mere number.—[Ed.]

When we use the word value in the sense of esteem, or urgency of desire, the feeling with which Oliver Twist must have regarded a few more mouthfuls when he "asked for more," the meaning of the word, as already explained, is identical with *degree of utility*, of which the dimension is U. Lastly, the *value in use* of Adam Smith, or the *total utility*, is the integral of U.dM, and has the dimensions MU. We may thus tabulate our results concerning the ambiguous uses of the word *value*—

Popular Expression of Meaning	Scientific Expression	Dimensions
(1) Value in use	Total Utility	MU
(2) Esteem, or Urgency of Desire for more	Final Degree of Utility .	U
(3) Purchasing Power	Ratio of Exchange .	M^0

Definition of Market

Before proceeding to the Theory of Exchange, it will be desirable to place beyond doubt the meanings of two other terms which I shall frequently employ.

By a *Market* I shall mean much what commercial men use it to express. Originally a market was a public place in a town where provisions and other objects were exposed for sale; but the word has been generalised, so as to mean any body of persons who are in intimate business relations and carry on extensive transactions in any commodity. A great city may contain as many markets as there are important branches of trade, and these markets may or may not be localised. The central point of a market is the

public exchange,—mart or auction rooms, where the traders agree to meet and transact business. In London, the Stock Market, the Corn Market, the Coal Market, the Sugar Market, and many others, are distinctly localised; in Manchester, the Cotton Market, the Cotton Waste Market, and others. But this distinction of locality is not necessary. The traders may be spread over a whole town, or region of country, and yet make a market, if they are, by means of fairs, meetings, published price lists, the post office, or otherwise, in close communication with each other. Thus, the common expression *Money Market* denotes no locality: it is applied to the aggregate of those bankers, capitalists, and other traders who lend or borrow money, and who constantly exchange information concerning the course of business.[1]

In Economics we may usefully adopt this term with a clear and well-defined meaning. By a market I shall mean two or more persons dealing in two or more commodities, whose stocks of those commodities and intentions of exchanging are known to all. It is also essential that the ratio of exchange between any

[1] I find that Cournot has long since defined the economic use of the word *market*, with admirable brevity and precision, but exactly to the same effect as the text above. He incidentally says in a footnote (*Récherches sur les Principes Mathématiques de la Théorie des Richesses*, Paris, 1838, p. 55), "On sait que les économistes entendent par *marché*, non pas un lieu déterminé ou se consomment les achats et les ventes, mais tout un territoire dont les parties sont unies par des rapports de libre commerce, en sorte que les prix s'y nivellent avec facilité et promptitude." [Translation by Bacon (Macmillan, 1897), p. 51 n.]

two persons should be known to all the others. It is only so far as this community of knowledge extends that the market extends. Any persons who are not acquainted at the moment with the prevailing ratio of exchange, or whose stocks are not available for want of communication, must not be considered part of the market. Secret or unknown stocks of a commodity must also be considered beyond reach of a market so long as they remain secret and unknown. Every individual must be considered as exchanging from a pure regard to his own requirements or private interests, and there must be perfectly free competition, so that any one will exchange with any one else for the slightest apparent advantage. There must be no conspiracies for absorbing and holding supplies to produce unnatural ratios of exchange. Were a conspiracy of farmers to withhold all corn from market, the consumers might be driven, by starvation, to pay prices bearing no proper relation to the existing supplies, and the ordinary conditions of the market would be thus overthrown.

The theoretical conception of a perfect market is more or less completely carried out in practice. It is the work of brokers in any extensive market to organise exchange, so that every purchase shall be made with the most thorough acquaintance with the conditions of the trade. Each broker strives to gain the best knowledge of the conditions of supply and demand, and the earliest intimation of any change. He is in communication with as many other traders

as possible, in order to have the widest range of information, and the greatest chance of making suitable exchanges. It is only thus that a definite market price can be ascertained at every moment, and varied according to the frequent news capable of affecting buyers and sellers. By the mediation of a body of brokers a complete *consensus* is established, and the stock of every seller or the demand of every buyer brought into the market. It is of the very essence of trade to have wide and constant information. A market, then, is theoretically perfect only when all traders have perfect knowledge of the conditions of supply and demand, and the consequent ratio of exchange; and in such a market, as we shall now see, there can only be one ratio of exchange of one uniform commodity at any moment.

So essential is a knowledge of the real state of supply and demand to the smooth procedure of trade and the real good of the community, that I conceive it would be quite legitimate to compel the publication of any requisite statistics. Secrecy can only conduce to the profit of speculators who gain from great fluctuations of prices. Speculation is advantageous to the public only so far as it tends to equalise prices; and it is, therefore, against the public good to allow speculators to foster artificially the inequalities of prices by which they profit. The welfare of millions, both of consumers and producers, depends upon an accurate knowledge of the stocks of cotton and corn; and it would, therefore, be no

unwarrantable interference with the liberty of the subject to require any information as to the stocks in hand. In Billingsgate fish market there was long ago a regulation to the effect that salesmen shall fix up in a conspicuous place every morning a statement of the kind and amount of their stock.[1] The same principle has long been recognised in the Acts of Parliament concerning the collection of statistics of the quantities and prices of corn sold in English market towns. More recently similar legislation has taken place as regards the cotton trade, in the Cotton Statistics Act of 1868. Publicity, whenever it can thus be enforced on markets by public authority, tends almost always to the advantage of everybody except perhaps a few speculators and financiers.

Definition of Trading Body

I find it necessary to adopt some expression for any number of people whose aggregate influence in a market, either in the way of supply or demand, we have to consider. By a *trading body* I mean, in the most general manner, any body either of buyers or sellers. The trading body may be a single individual in one case; it may be the whole inhabitants of a continent in another; it may be the individuals of a trade diffused through a country in a third. England and North America will be trading bodies if we are considering the corn we receive from

[1] Waterston's *Cyclopædia of Commerce*, ed. 1846, p. 466.

America in exchange for iron and other goods. The continent of Europe is a trading body as purchasing coal from England. The farmers of England are a trading body when they sell corn to the millers, and the millers both when they buy corn from the farmers and sell flour to the bakers.

We must use the expression with this wide meaning, because the principles of exchange are the same in nature, however wide or narrow may be the market considered. Every trading body is either an individual or an aggregate of individuals, and the law, in the case of the aggregate, must depend upon the fulfilment of law in the individuals. We cannot usually observe any precise and continuous variation in the wants and deeds of an individual, because the action of extraneous motives, or what would seem to be caprice, overwhelms minute tendencies. As I have already remarked (p. 15), a single individual does not vary his consumption of sugar, butter, or eggs from week to week by infinitesimal amounts, according to each small change in the price. He probably continues his ordinary consumption until accident directs his attention to a rise in price, and he then, perhaps, discontinues the use of the articles altogether for a time. But the aggregate, or what is the same, the average consumption, of a large community will be found to vary continuously or nearly so. The most minute tendencies make themselves apparent in a wide average. Thus, our laws of Economics will be theoretically true in the case of

individuals, and practically true in the case of large aggregates; but the general principles will be the same, whatever the extent of the trading body considered. We shall be justified, then, in using the expression with the utmost generality.

It should be remarked, however, that the economic laws representing the conduct of large aggregates of individuals will never represent exactly the conduct of any one individual. If we could imagine that there were a thousand individuals all exactly alike in regard to their demand for commodities, and their capabilities of supplying them, then the average laws of supply and demand deduced from the conduct of such individuals would agree with the conduct of any one individual. But a community is composed of persons differing widely in their powers, wants, habits, and possessions. In such circumstances the average laws applying to them will come under what I have elsewhere[1] called the "Fictitious Mean," that is to say, they are numerical results which do not pretend to represent the character of any existing thing. But average laws would not on this account be less useful, if we could obtain them; for the movements of trade and industry depend upon averages and aggregates, not upon the whims of individuals.

The Law of Indifference

When a commodity is perfectly uniform or homogeneous in quality, any portion may be indifferently

[1] *Principles of Science*, 1st ed., vol. i. p. 422; 3d ed., p. 363.

used in place of an equal portion: hence, in the same market, and at the same moment, all portions must be exchanged at the same ratio. There can be no reason why a person should treat exactly similar things differently, and the slightest excess in what is demanded for one over the other will cause him to take the latter instead of the former. In nicely balanced exchanges it is a very minute scruple which turns the scale and governs the choice. A minute difference of quality in a commodity may thus give rise to preference, and cause the ratio of exchange to differ. But where no difference exists at all, or where no difference is known to exist, there can be no ground for preference whatever. If, in selling a quantity of perfectly equal and uniform barrels of flour, a merchant arbitrarily fixed different prices on them, a purchaser would of course select the cheaper ones; and where there was absolutely no difference in the thing purchased, even an excess of a penny in the price of a thing worth a thousand pounds would be a valid ground of choice. Hence follows what is undoubtedly true, with proper explanations, that *in the same open market, at any one moment, there cannot be two prices for the same kind of article.* Such differences as may practically occur arise from extraneous circumstances, such as the defective credit of the purchasers, their imperfect knowledge of the market, and so on.

The principle above expressed is a general law of the utmost importance in Economics, and I propose

to call it *The Law of Indifference*, meaning that, when two objects or commodities are subject to no important difference as regards the purpose in view, they will either of them be taken instead of the other with perfect indifference by a purchaser. Every such act of indifferent choice gives rise to an equation of degrees of utility, so that in this principle of indifference we have one of the central pivots of the theory.

Though the price of the same commodity must be uniform at any one moment, it may vary from moment to moment, and must be conceived as in a state of continual change. Theoretically speaking, it would not usually be possible to buy two portions of the same commodity *successively* at the same ratio of exchange, because, no sooner would the first portion have been bought than the conditions of utility would be altered. When exchanges are made on a large scale, this result will be verified in practice.[1] If a wealthy person invested £100,000 in the funds in the morning, it is hardly likely that the operation could be repeated in the afternoon at the same price. In any market, if a person goes on buying largely, he will ultimately

[1] It is, I believe, verified in the New York Stock Markets, where it is the practice to sell Stocks by auction in successive lots, without disclosing the total amount to be put up. When the amount offered begins to exceed what was expected, then each successive lot brings a less price, and those who bought the earlier lots suffer. But if the amount offered is small, the early buyers have the advantage. Such an auction sale only exhibits in miniature what is constantly going on in the markets generally on a large scale.

raise the price against himself. Thus it is apparent that extensive purchases would best be made gradually, so as to secure the advantage of a lower price upon the earlier portions. In theory this effect of exchange upon the ratio of exchange must be conceived to exist in some degree, however small may be the purchases made. Strictly speaking, the ratio of exchange at any moment is that of dy to dx, of an infinitely small quantity of one commodity to the infinitely small quantity of another which is given for it. The ratio of exchange is really a differential coefficient. The quantity of any article purchased is a function of the price at which it is purchased, and the ratio of exchange expresses the rate at which the quantity of the article increases compared with what is given for it.

We must carefully distinguish, at the same time, between the Statics and Dynamics of this subject. The real condition of industry is one of perpetual motion and change. Commodities are being continually manufactured and exchanged and consumed. If we wished to have a complete solution of the problem in all its natural complexity, we should have to treat it as a problem of motion—a problem of dynamics. But it would surely be absurd to attempt the more difficult question when the more easy one is yet so imperfectly within our power. It is only as a purely statical problem that I can venture to treat the action of exchange. Holders of commodities will be regarded not as continuously passing on these commodities in

streams of trade, but as possessing certain fixed amounts which they exchange until they come to equilibrium.

It is much more easy to determine the point at which a pendulum will come to rest than to calculate the velocity at which it will move when displaced from that point of rest. Just so, it is a far more easy task to lay down the conditions under which trade is completed and interchange ceases, than to attempt to ascertain at what rate trade will go on when equilibrium is not attained.

The difference will present itself in this form: dynamically we could not treat the ratio of exchange otherwise than as the ratio of dy and dx, infinitesimal quantities of commodity. Our equations would then be regarded as differential equations, which would have to be integrated. But in the statical view of the question we can substitute the ratio of the finite quantities y and x. Thus, from the self-evident principle, stated on pp. 91-2, that there cannot, in the same market, at the same moment, be two different prices for the same uniform commodity, it follows that *the last increments in an act of exchange must be exchanged in the same ratio as the whole quantities exchanged.* Suppose that two commodities are bartered in the ratio of x for y; then every m^{th} part of x is given for the m^{th} part of y, and it does not matter for which of the m^{th} parts. No part of the commodity can be treated differently from any other part. We may carry this division to an indefinite

extent by imagining m to be constantly increased, so that, at the limit, even an infinitely small part of x must be exchanged for an infinitely small part of y, in the same ratio as the whole quantities. This result we may express by stating that the increments concerned in the process of exchange must obey the equation

$$\frac{dy}{dx}=\frac{y}{x}.$$

The use which we shall make of this equation will be seen in the next section.

The Theory of Exchange

The keystone of the whole Theory of Exchange, and of the principal problems of Economics, lies in this proposition—*The ratio of exchange of any two commodities will be the reciprocal of the ratio of the final degrees of utility of the quantities of commodity available for consumption after the exchange is completed.* When the reader has reflected a little upon the meaning of this proposition, he will see, I think, that it is necessarily true, if the principles of human nature have been correctly represented in previous pages.

Imagine that there is one trading body possessing only corn, and another possessing only beef. It is certain that, under these circumstances, a portion of the corn may be given in exchange for a portion of the beef with a considerable increase of utility. How are we to determine at what point the exchange will

cease to be beneficial? This question must involve both the ratio of exchange and the degrees of utility. Suppose, for a moment, that the ratio of exchange is approximately that of ten pounds of corn for one pound of beef: then if, to the trading body which possesses corn, ten pounds of corn are less useful than one of beef, that body will desire to carry the exchange further. Should the other body possessing beef find one pound less useful than ten pounds of corn, this body will also be desirous to continue the exchange. Exchange will thus go on until each party has obtained all the benefit that is possible, and loss of utility would result if more were exchanged. Both parties, then, rest in satisfaction and equilibrium, and the degrees of utility have come to their level, as it were.

This point of equilibrium will be known by the criterion, that an infinitely small amount of commodity exchanged in addition, at the same rate, will bring neither gain nor loss of utility. In other words, if increments of commodities be exchanged at the established ratio, their utilities will be equal for both parties. Thus, if ten pounds of corn were of exactly the same utility as one pound of beef, there would be neither harm nor good in further exchange at this ratio.

It is hardly possible to represent this theory completely by means of a diagram, but the accompanying figure may, perhaps, render it clearer. Suppose the line pqr to be a small portion of the

curve of utility of one commodity, while the broken line $p'qr'$ is the like curve of another commodity which has been reversed and superposed on the other. Owing to this reversal, the quantities of the first commodity are measured along the base line from a towards b, whereas those of the second must be measured in the opposite direction. Let units of both commodities be represented by equal lengths: then the little line $a'a$ indicates an increase of the first commodity, and a decrease of the second. Assume the ratio of exchange to be that of unit for unit, or

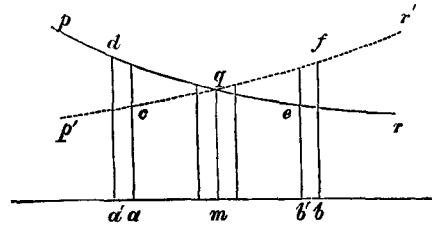

FIG. 5.

1 to 1: then, by receiving the commodity $a'a$ the person will gain the utility ad, and lose the utility $a'c$; or he will make a net gain of the utility corresponding to the mixtilinear figure cd. He will, therefore, wish to extend the exchange. If he were to go up to the point b', and were still proceeding, he would, by the next small exchange, receive the utility be, and part with $b'f$; or he would have a net loss of ef. He would, therefore, have gone too far; and it is pretty obvious that the point of intersection, q, defines the place where he would stop with the greatest

advantage. It is there that a net gain is converted into a net loss, or rather where, for an infinitely small quantity, there is neither gain nor loss. To represent an infinitely small quantity, or even an exceedingly small quantity, on a diagram is, of course, impossible; but on either side of the line mq I have represented the utilities of a small quantity of commodity more or less, and it is apparent that the net gain or loss upon the exchange of these quantities would be trifling.

Symbolic Statement of the Theory

To represent this process of reasoning in symbols, let Δx denote a small increment of corn, and Δy a small increment of beef exchanged for it. Now our Law of Indifference comes into play. As both the corn and the beef are homogeneous commodities, no parts can be exchanged at a different ratio from other parts in the same market: hence, if x be the whole quantity of corn given for y the whole quantity of beef received, Δy must have the same ratio to Δx as y to x; we have then,

$$\frac{\Delta y}{\Delta x}=\frac{y}{x}, \quad \text{or} \quad \Delta y=\frac{y}{x}\Delta x.$$

In a state of equilibrium, the utilities of these increments must be equal in the case of each party, in order that neither more nor less exchange would be desirable. Now the increment of beef, Δy, is $\frac{y}{x}$ times as great as the increment of corn, Δx, so that,

in order that their utilities shall be equal, the degree of utility of beef must be $\frac{x}{y}$ times as great as the degree of utility of corn. Thus we arrive at the principle that *the degrees of utility of commodities exchanged will be in the inverse proportion of the magnitudes of the increments exchanged.*

Let us now suppose that the first body, A, originally possessed the quantity a of corn, and that the second body, B, possessed the quantity b of beef. As the exchange consists in giving x of corn for y of beef, the state of things after exchange will be as follows:—

A holds $a-x$ of corn, and y of beef,
B holds x of corn, and $b-y$ of beef.

Let $\phi_1(a-x)$ denote the final degree of utility of corn to A, and $\phi_2 x$ the corresponding function for B. Also let $\psi_1 y$ denote A's final degree of utility for beef, and $\psi_2(b-y)$ B's similar function. Then, as explained on p. 96, A will not be satisfied unless the following equation holds true:—

$$\phi_1(a-x)\,.\,dx=\psi_1 y\,.\,dy;$$
$$\text{or} \quad \frac{\phi_1(a-x)}{\psi_1 y}=\frac{dy}{dx}.$$

Hence, substituting for the second member by the equation given on p. 95, we have

$$\frac{\phi_1(a-x)}{\psi_1 y}=\frac{y}{x}.$$

What holds true of A will also hold true of B,

mutatis mutandis. He must also derive exactly equal utility from the final increments, otherwise it will be for his interest to exchange either more or less, and he will disturb the conditions of exchange. Accordingly the following equation must hold true :—

$$\psi_2(b-y)\,.\,dy = \phi_2 x\,.\,dx\,;$$

or, substituting as before,

$$\frac{\phi_2 x}{\psi_2(b-y)} = \frac{y}{x}.$$

We arrive, then, at the conclusion, that whenever two commodities are exchanged for each other, and *more or less can be given or received in infinitely small quantities*, the quantities exchanged satisfy two equations, which may be thus stated in a concise form—

$$\frac{\phi_1(a-x)}{\psi_1 y} = \frac{y}{x} = \frac{\phi_2 x}{\psi_2(b-y)}.$$

The two equations are sufficient to determine the results of exchange; for there are only two unknown quantities concerned, namely, x and y, the quantities given and received.

A vague notion has existed in the minds of economical writers, that the conditions of exchange may be expressed in the form of an equation. Thus, J. S. Mill has said :[1] "The idea of a *ratio*, as between demand and supply, is out of place, and has no concern in the matter : the proper mathematical analogy is that of an *equation.* Demand and supply, the quantity demanded and the quantity supplied, will be

[1] *Principles of Political Economy,* book iii., chap. ii., sec. 4.

made equal." Mill here speaks of an equation as only a proper mathematical *analogy.* But if Economics is to be a real science at all, it must not deal merely with analogies; it must reason by real equations, like all the other sciences which have reached at all a systematic character. Mill's equation, indeed, is not explicitly the same as any at which we have arrived above. His equation states that the quantity of a commodity given by A is equal to the quantity received by B. This seems at first sight to be a mere truism, for this equality must necessarily exist if any exchange takes place at all. The theory of value, as expounded by Mill, fails to reach the root of the matter, and show how the amount of demand or supply is caused to vary. And Mill does not perceive that, as there must be two parties and two quantities to every exchange, there must be two equations.

Nevertheless, our theory is perfectly consistent with the laws of supply and demand; and if we had the functions of utility determined, it would be possible to throw them into a form clearly expressing the equivalence of supply and demand. We may regard x as the quantity demanded on one side and supplied on the other; similarly, y is the quantity supplied on the one side and demanded on the other. Now, when we hold the two equations to be simultaneously true, we assume that the x and y of one equation equal those of the other. The laws of supply and demand are thus a result of what seems to me the true theory of value or exchange.

Analogy to the Theory of the Lever

I have heard objections made to the general character of the equations employed in this book. It is remarked that the equations in question continually involve infinitesimal quantities, and yet they are not treated as differential equations usually are, that is integrated. There is, indeed, no reason why the process of integration should not be applied when it is required, and I will here show that the equations employed do not differ in general character from those which are really treated in many branches of physical science. Whenever, in fact, we deal with continuously varying quantities, the ultimate equations must lie between infinitesimals. The process of integration, if I understand the matter aright, only ascertains other equations, the truth of which follows from the fundamental differential equation.

The mode in which mechanics is usually treated in elementary work tends to disguise the real foundation of the science which is to be found in the so-called *theory of virtual velocities.* Let us take the description of the lever of the first order as it is given in some of the best modern elementary works, as, for instance, in Mr. Magnus's *Lessons in Elementary Mechanics*, p. 128. We here read as follows:—

"Let AB be a lever turning freely about C, the fulcrum, and let P be the force applied at A, and W the force exerted, or resistance overcome, or weight

raised at B. Suppose the lever turned through the angle ACA', then the work done by P equals $P \times$ arc AA', and work done by W equals $W \times$ arc BB', if P and W act perpendicularly to the arm. Therefore, by the law of energy,

$$P \times AA' = W \times BB', \text{ and since } \frac{AA'}{BB'} = \frac{AC}{BC} \text{ we have}$$

$$P \times AC = W \times BC,$$

or, $P \times$ its arm $= W \times$ its arm."

Now, in such a statement as this, we seem to be dealing with plain finite quantities, and there is no apparent difficulty in the matter. In reality the difficulty is only disguised by assuming that P and W act perpendicularly to the arm through finite arcs. This condition is, indeed, carried out with approximate exactness in the problem of the wheel and axle,[1] which may be regarded as combining together an infinite series of straight levers, coming successively into operation. In this machine, therefore, the weights, roughly speaking, always act perpendicularly to arms of invariable length. But, in the generality of cases of the lever, the theory is only true for infinitely small displacements, and no sooner has the lever begun to move through any finite arc AA', than it ceases to be exactly true that the work done by P equals $P \times$ arc AA'. Nevertheless, the theory is quite correct as applied to the lever considered statically, that is, as in a state of

[1] See Magnus's *Lessons*, sec. 91.

rest and equilibrium, because the finite arcs of displacement, when it really is displaced, are exactly proportional to the infinitely small arcs, known as virtual velocities, through which it would be displaced, if instead of being at rest, it suffered an infinitely small displacement.

It is curious, moreover, that, when we take the theory of the lever treated according to the principle of virtual velocities, we get equations exactly similar in form to those of the theory of value as established above. The general principle of virtual velocities is to the effect that, if any number of forces be in equilibrium at one or more points of a rigid body, and if this body receive an infinitely small displacement, the algebraic sum of the products of each force into its displacement is equal to zero. In the case of a lever of the first order, this amounts to saying that one force multiplied into its displacement will be neutralised by the other force multiplied into its *negative* displacement. But inasmuch as the displacements are exactly proportional to the lengths of the arms of the lever, we obtain as a derivative equation, that the forces multiplied each by its own arm are equal to each other. No doubt in the quotation given above, $P \times AC = W \times BC$ is an equation between finite quantities; but the real equation derived immediately from the principle of virtual velocities, is $P \times AA' = W \times BB'$, in which P and W are finite, but AA' and BB' are in strictness infinitely small displacements. Let us write this

equation in the form $\frac{W}{P} = \frac{AA'}{BB'}$; then as we also have $\frac{AA'}{BB'} = \frac{AC}{BC}$ we can substitute; hence $\frac{W}{P} = \frac{AC}{BC}$.

I dwell upon this matter at some length because we here have exactly the forms of the equations of exchange. As we have seen, the original equation is of the general form $\frac{\phi x}{\psi y} = \frac{dy}{dx}$, where ψy and ϕx represent finite expressions for the degrees of utility of the commodities Y and X, as regards some individual, and dy and dx are infinitesimal quantities of those commodities exchanged. But these infinitesimals may in this case at least be eliminated, because, in virtue of the Law of Indifference, they are exactly proportional to the whole finite quantities exchanged. Hence for $\frac{dy}{dx}$ we substitute $\frac{y}{x}$. We may write the equations one below the other, so as to make the analogy visible—thus

$$\frac{W}{P} = \frac{AA'}{BB'} = \frac{AC}{BC}$$

$$\frac{\phi x}{\psi y} = \frac{dy}{dx} = \frac{y}{x}.$$

To put this analogy of the theories of exchange and of the lever in the clearest possible light, I give below a diagram, in which the several economic qualities are represented by the parts of the diagram to which they correspond or are proportional.

Now in statical problems no such process as

integration is applicable. The equation lies actually between imaginary infinitesimal quantities, and there is no effect to be summed up. Yet there is no statical problem which is not subject to the principle of virtual velocities, and Poisson, in his *Traité de*

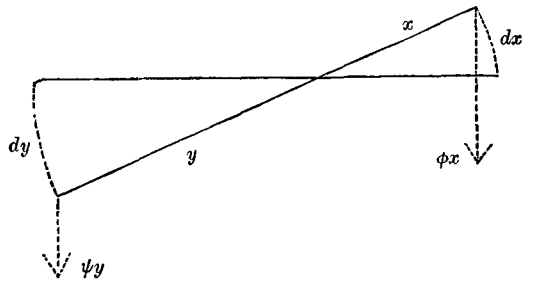

FIG. 6.

Mécanique, which commences with statical theorems, asserts explicitly,[1] "Dans cet ouvrage, j'emploierai exclusivement la méthode *des infiniment petits.*"

Impediments to Exchange

We have hitherto treated the theory of exchange as if the action of exchange could be carried on without trouble or cost. In reality, the cost of conveyance is almost always of importance, and it is sometimes the principal element in the question. To the cost of mere transport must be added a variety of charges of brokers, agents, packers, dock, harbour, light dues, etc., together with any customs duties imposed either on the importation or exportation of commodities. All these charges, whether necessary

[1] Seconde Édition, Paris, 1833, sec. 12, vol. i. p. 14.

or arbitrary, are so many impediments to commerce, and tend to reduce its advantages. The effect of any one such charge, or of the aggregate of the costs of exchange, can be represented in our formulæ in a very simple manner.

In whatever modes the charges are payable, they may be conceived as paid by the surrender on importation of a certain fraction of the commodity received; for the amount of the charges will usually be proportional to the quantity of goods, and, if expressed in money, can be considered as turned into commodity.

Thus, if A gives x in exchange, this is not the quantity received by B; a part of x is previously subtracted, so that B receives say mx, which is less than x, and the terms of exchange must be adjusted on his part so as to agree with this condition. Hence the second equation will be

$$\frac{y}{mx} = \frac{\phi_2(mx)}{\psi_2(b-y)}.$$

Again, A, though giving x, will not receive the whole of y; but say ny, so that his equation similarly will be

$$\frac{\phi_1(a-x)}{\psi_1(ny)} = \frac{ny}{x}.$$

The result is, that there is not one ratio of exchange, but two ratios; and the more these differ, the less advantage will there be in exchange. It is obvious that A has either to remain satisfied with less of the second commodity than before, or has to give more

of his own in purchasing it. By an obvious transfer of the factors m and n we may state the equations of impeded exchange in the concise form—

$$\frac{\phi_1(a-x)}{n.\psi_1(ny)} = \frac{y}{x} = \frac{m.\phi_2(mx)}{\psi_2(b-y)}.$$

Illustrations of the Theory of Exchange

As stated above, the Theory of Exchange may seem to be of a somewhat abstract and perplexing character; but it is not difficult to find practical illustrations which will show how it is verified in the actual working of a great market. The ordinary laws of supply and demand, when properly stated, are the practical manifestation of the theory. Considerable discussion has taken place concerning these laws, in consequence of Mr. W. T. Thornton's writings upon the subject in the *Fortnightly Review*, and in his work on the *Claims of Labour*. Mill, although he had previously declared the Theory of Value to be complete and perfect (see p. 76), was led by Mr. Thornton's arguments to allow that modification was required.

For my own part, I think that most of Mr. Thornton's arguments are beside the question. He suggests that there are no regular laws of supply and demand, because he adduces certain cases in which no regular variation can take place. Those cases might be indefinitely multiplied, and yet the laws in question would not be touched. Of course, laws which assume

a continuity of variation are inapplicable where continuous variation is impossible. Economists can never be free from difficulties unless they will distinguish between a theory and the *application of a theory*. Because, in retail trade, in English or Dutch auction, or other particular modes of traffic, we cannot at once observe the operation of the laws of supply and demand, it is not in the least to be supposed that those laws are false. In fact, Mr. Thornton seems to allow that, if prospective demand and supply are taken into account, they become substantially true. But, in the actual working of any market, the influence of future events should never be neglected, neither by a merchant nor an economist.

Though Mr. Thornton's objections are mostly beside the question, his remarks have served to show that the action of the laws of supply and demand was inadequately explained by previous economists. What constitutes the demand and the supply was not carefully enough investigated. As Mr. Thornton points out, there may be a number of persons willing to buy; but if their highest offer is ever so little short of the lowest price which the seller is willing to take, their influence is *nil*. If in an auction there are ten people willing to buy a horse at £20, but not higher, their demand instantly ceases when any one person offers £21. I am inclined not only to accept such a view, but to carry it further. Any change in the price of an article will be determined not with regard to the large numbers who might or might not

exceeds the desire of possession, including all the motives for exertion.

We must consider the duration of labour as measured by the number of hours' work per day. The alternation of day and night on the earth has rendered man essentially periodic in his habits and actions. In a natural and wholesome condition a man should return each twenty-four hours to exactly the same state; at any rate, the cycle should be closed within the seven days of the week. Thus the labourer must not be supposed to be either increasing or diminishing his normal strength. But the theory might also be made to apply to cases where special exertion is undergone for many days or weeks in succession, in order to complete work, as in collecting the harvest. Adequate motives may lead to and warrant overwork, but, if long continued, excessive labour reduces the strength and becomes insupportable; and the longer it continues the worse it is, the law being somewhat similar to that of periodic labour.

Symbolic Statement of the Theory

In attempting to represent these conditions of labour with accuracy, we shall find that there are no less than four quantities concerned; let us denote them as follows:—

t = time, or duration of labour;

l = amount of labour, as meaning the aggregate balance of pain accompanying it, irrespective of the produce;

x = amount of commodity produced;

u = total utility of that commodity.

The amount of commodity produced will be very different in different cases. In any one case the rate of production will be determined by dividing the whole quantity produced by the time of production, provided that the rate of production has been uniform; it will then be $\frac{x}{t}$. But if the rate of production be variable, it can only be determined at any moment by comparing a small quantity of produce with the small portion of time occupied in its production. More strictly speaking, we must ascertain the ratio of an infinitely small quantity of produce to the corresponding infinitely small portion of time. Thus *the rate of production* is properly denoted by $\frac{\Delta x}{\Delta t}$, or at the limit by $\frac{dx}{dt}$.

Again, the degree of painfulness of labour would be $\frac{l}{t}$ if it remained invariable; but as it is highly variable, we must again compare small increments, and $\frac{\Delta l}{\Delta t}$, or, at the limit, $\frac{dl}{dt}$ correctly represents the *degree of painfulness of labour*. But we must also take into account the fact that the utility of commodity is not constant. If a man works regularly twelve hours a day, he will produce more commodity than in ten hours; therefore the final degree of utility of his commodity, whether he consume it himself or