Human capital formation, public debt and economic growth

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Abstract

This paper presents an endogenous growth model with human capital, where human capital formation is the result of public education. The government finances expenditures in the schooling sector by the tax revenue and by public deficit. In addition, the government sets the primary surplus such that it is a positive linear function of public debt which guarantees that public debt is sustainable. The paper analyzes the structure of the growth model and derives implications of public debt. Further, a sensitivity analysis of the dynamics of the model is presented and it is demonstrated that for certain parameter values the model may produce multiple balanced growth paths and sustained cycles.

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1 Introduction

One force of sustained per capita growth in endogenous growth models is human capital. The seminal papers in this respect are the contributions by Uzawa (1965) and by Lucas (1988). There, the representative individual decides how much of his available time is spent for producing physical output and how much is used for the formation of human capital. Rebelo (1991) extended this class of models by assuming that both physical capital and human capital enter the production process of human capital, in contrast to the model by Uzawa and Lucas who posit that human capital formation is the result of human capital input alone.

However, neither of theses models allows for public spending in the process of human capital formation. Contributions, which take into account that the public sector can stimulate the formation of human capital by devoting public resources to schooling, are for example Glomm and Ravikumar (1992), Ni and Wang (1994), Beauchemin (2001) or Blankenau and Simpson (2004). In those contributions, human capital accumulation results either from both private and public services, as in Glomm and Ravikumar and in Blankenau, or from public spending alone, as in Ni and Wang and in Beauchemin.

As concerns the empirical relevance of human capital, there is evidence that education is positively correlated with income growth. At the microeconomic level the positive correlation seems to be quite robust. On the macroeconomic level the findings are more fragile (cf. Krueger and Lindahl, 2001) which, however, may be due to measurement errors. So, Krueger and Lindahl demonstrate that cross-country regressions indicate that the change in education is positively correlated with economic growth if measurement errors are accounted for. Further, Levine and Renelt (1992) have shown that human capital, measured by the secondary enrollment rate, is a robust variable in growth regressions, so that building endogenous growth models with human capital as the engine of growth seems to be justified.

When the government can influence the process of human capital formation by adequate expenditures, it may finance these measures by the tax revenue and by public deficits. As concerns deficit finance of productive public spending in endogenous growth models with an infinitely lived representative individual, one realizes that a deficit financed increase in public spending leads to higher long-run growth (see e.g. Turnovsky, 1995, p. 418). The reason for this outcome is that deficit finance of the government does not have any distortions in the model with an infinitely lived individual. Consequently, the growth stimulating effect of higher productive spending dominates and leads to a higher balanced growth rate. Of course, this result is independent of whether the government finances human capital or public infrastructure as long as these variables foster economic growth.

However, this outcome changes if budgetary regimes are introduced into the model as in Greiner and Semmler (2000) and in Gosh and Mourmouras (2004). Budgetary regimes are rules to which the government must stick and they imply feedback effects of higher public debt, so that a deficit financed increase in public investment may lead to a smaller growth rate in the long-run. Although intuitive and realistic, integrating such budgetary regimes in economic models may seem to be ad hoc.

Another way of bringing a feedback effect of higher public debt into economic models is to assume that the primary surplus is a positive linear function of public debt, as ratio to GDP respectively. Then, deficits today lead to a rise of the primary surplus in the future, implying that the government has less scope for other, productive, spending. The motivation for this rule was provided by Bohn (1995, 1998) who showed that this policy guarantees that public debt remains sustainable and who also found empirical evidence for this rule in the US.

In this paper, we present an endogenous growth model with human capital formation where the government plays the decisive role by financing teachers and other school expenditures. In addition, we posit that the primary surplus-GDP ratio is a positive linear function of the debt-GDP ratio so that public debt is sustainable. We take the number of students as exogenously given because we focus on the effects of public debt on human capital formation given a certain number of students. We are interested in growth effects and in the dynamics of our model.

The rest of the paper is organized as follows. In the next section, we present the structure of our growth model. In section 3, we derive implications of our model as concerns public debt and economic growth and section 4, finally, concludes.

2 The structure of the growth model

Our economy consists of three sectors: A household sector which receives labour income and income from its saving, a productive sector and the government. First, we describe the household and the productive sector.

2.1 The household and the productive sector

Overall population in our economy is composed of a stock of students, S, and of a stock of employees, L, who constitute the active labour force and produce goods or are hired as teachers. At each point of time a certain number of students, which is determined exogenously, enters the stock of students and a certain number of students becomes employees. We assume that the number of students becoming employees just equals the number of new students so that the overall stock of students is constant. Further, the number of students becoming employees equals the number of employees leaving the active labour force, so that the active labour force and, thus, total population are constant, too.

The household sector is represented by one household which maximizes the discounted stream of utility resulting from consumption, C,[1] over an infinite time horizon subject to its budget constraint. The utility function is assumed to be logarithmic, $U(C) = \ln C$, and labour, L, is inelastically supplied. The maximization problem, then, can be written

[1]We omit the time argument t if no ambiguity arises.

as

$$\max_{C} \int_{0}^{\infty} e^{-\rho t} \ln C \, dt, \tag{1}$$

subject to

$$(1-\tau)(wL + rW) = \dot{W} + C. \tag{2}$$

ρ is the subjective discount rate, w is the wage rate and r is the interest rate. $W \equiv B + K$ denotes assets which are equal to public debt, B, and physical capital, K. $\tau \in (0,1)$ is the income tax rate. The dot gives the derivative with respect to time and we neglect depreciation of private capital.

To solve this problem we formulate the current-value Hamiltonian which is written as

$$\mathcal{H} = \ln C + \lambda((1-\tau)(wL + rW) - C) \tag{3}$$

Necessary optimality conditions are given by

$$C^{-1} = \lambda \tag{4}$$

$$\dot{\lambda} = \rho\lambda - \lambda(1-\tau)\, r \tag{5}$$

If the transversality condition $\lim_{t\to\infty} e^{-\rho t} W/C = 0$ holds, which is fulfilled for a time path on which assets grow at the same rate as consumption, the necessary conditions are also sufficient.

The productive sector is represented by one firm which behaves competitively and which maximizes static profits. The production function of the firm is given by

$$Y = K^{1-\alpha}(u h_c L)^{\alpha}, \tag{6}$$

with $0 < \alpha < 1$. $(1-\alpha)$ is the private capital share and α gives the labour share. h_c denotes human per capita capital which is labour augmenting and u is that part of the labour force employed in the final goods sector. Profit maximization yields

$$w = \alpha(uL)^{-1} Y \tag{7}$$

$$r = (1-\alpha) K^{-1} Y \tag{8}$$

Using (4), (5), (6) and (8), which must hold in equilibrium, the growth rate of consumption is derived as

$$\frac{\dot{C}}{C} = -\rho + (1-\tau)(1-\alpha)K^{-\alpha}(uh_c L)^{\alpha}. \tag{9}$$

2.2 Human capital formation

Human capital in our economy is produced in the schooling sector where an exogenously given number of students is educated. The government hires $(1-u)$ of the active labour force as teachers whom it pays the competitive wage rate. We assume that the government decides about the part of the active labour force employed in the schooling sector and, thus, determines the part of the labour force in the final goods sector. Additionally, the government uses public resources for education in the schooling sector, like expenditures for books and other teaching material, which is an input in the process of human capital formation, too. Thus, the input in the schooling sector is composed of teachers and of schooling expenditures and we assume decreasing returns to scale. The evolution of per capita human capital, then, is a function of teachers per student and of school spending per student.

It should be noted that human capital, which is embodied in students, becomes available to the whole active labour force in the economy, once students become employees. The reason for this assumption is to be seen in spill-over effects of knowledge, which leads to a diffusion of knowledge among the labour force. At first sight, this seems to be a strong assumption. But if one takes into account that in reality newly hired employees interact with existing staff and both learn from each other, this assumption becomes plausible.

As concerns the production function for human capital formation we assume a Cobb-Douglas specification. Normalizing labour to one, Ł $\equiv 1$, which holds from now on, the differential equation describing the change in human per capita capital can be written as

$$\dot{h}_c = \epsilon((1-u)h_c)^{\gamma}(I_E)^{1-\gamma}/S, \tag{10}$$

with I_E public resources used in the schooling sector, $\epsilon > 0$ a technology parameter and $\gamma \in (0,1)$ is the elasticity of human capital formation with respect to teachers.

2.3 The government

The government in our economy receives tax revenues from income taxation and has revenues from issuing government bonds it then uses for the remuneration of the teachers, for public spending in the schooling sector and for interest payments on public debt. Thus, the period budget constraint of the government is given by

$$T + \dot{B} = I_E + (1-u)w + rB, \tag{11}$$

with T denoting tax revenue.

Further, the government fixes the primary surplus to GDP ratio such that it is a positive linear function of the debt-GDP ratio. The motivation for this rule is that it guarantees sustainability of public debt. To see this, recall that a sustainable policy requires that the government does not play a Ponzi game, that is the following condition must hold (see e.g. Blanchard and Fischer, 1989, ch. 2),

$$\lim_{t \to \infty} e^{-\int_0^t r(\tau)d\tau} B(t) = 0 \leftrightarrow B(0) = \int_0^t e^{-\int_0^\tau r(\mu)d\mu} P(\tau)d\tau, \tag{12}$$

with P denoting the primary surplus.

Equation (12) is the present-value borrowing constraint and we call a path of public debt which satisfies this constraint sustainable. It states that public debt at time zero must equal future present-value surpluses and rules out that the government plays a Ponzi game.

Now, assume that the ratio of the primary surplus to GDP ratio is a positive linear function of the debt-GDP ratio and of a constant (cf. Bohn, 1995, 1998). The primary surplus ratio, then, can be written as

$$\frac{P}{Y} = \phi + \beta \, \frac{B}{Y} \, , \tag{13}$$

where $\phi, \beta \in \mathbb{R}$ are constants and $\beta > 0$ holds. It should be noted that β determines how strong the primary surplus reacts to changes in public debt and, therefore, can be considered as a reaction coefficient of the government to variations in public debt. ϕ determines whether the level of the primary surplus rises or falls with an increase in GDP.

Using (13), the differential equation describing the evolution of public debt can be written as

$$\dot{B} = (r - \beta)B - \phi Y. \tag{14}$$

Solving this differential equation and multiplying both sides by $e^{-\int_0^t r(\tau)d\tau}$, to get the present value of public debt, yields

$$e^{-\int_0^t r(\tau)d\tau} B(t) = e^{-\beta t} \left(B(0) - \phi Y(0) \int_0^t e^{\beta\tau - \int_0^\tau (r(\mu) - g_y(\mu))d\mu} d\tau \right), \tag{15}$$

with $B(0)$ public debt at time $t = 0$ and g_y the growth rate of gross domestic income.

First, we state that for $r < g_y$ the intertemporal budget constraint is irrelevant because in this case the economy is dynamically inefficient implying that the government can play a Ponzi game. Therefore, we only consider the case $r > g_y$.

Writing equation (15) as

$$e^{-\int_0^t r(\tau)d\tau} B(t) = e^{-\beta t}\, B(0) - \phi Y(0)\, \frac{\int_0^t e^{\beta\tau}\, e^{-\int_0^\tau (r(\mu) - g_y(\mu))d\mu} d\tau}{e^{\beta t}}\, . \tag{16}$$

shows that $\beta > 0$ is a necessary condition for $\lim_{t\to\infty} e^{-\int_0^t r(\tau)d\tau} B(t) = 0$, i.e. for the present value of public debt to converge to zero for $t \to \infty$.

If the numerator in the second expression in (16) remains finite, implying that $\int_0^\tau (r(\mu) - g_y(\mu))d\mu$ converges to infinity, the second term converges to zero. If the numerator in the second expression in (16) becomes infinite, l'Hôpital gives the limit as $e^{-\int_0^t (r(\mu) - g_y(\mu))d\mu}/\beta$. This shows that $\beta > 0$ and $\lim_{t\to\infty} \int_0^t (r(\mu) - g_y(\mu))d\mu = \infty$ are sufficient for sustainability of public debt.

These considerations demonstrate that the intertemporal budget constraint of the government is fulfilled if the ratio of the primary surplus to GDP is a positive linear function of the debt ratio, which can also be observed for economies in the real world. Therefore, we posit that the government sets the primary surplus according to (13) implying that public debt is sustainable.

From an economic point of view, this assumption brings a feedback effect of higher government debt into the model. If the government increases public debt, for whatever reasons, it must raise the primary surplus so that fiscal policy remains sustainable. This, however, means that more resources must be used for the debt service implying that the government has less scope for other types of spending, e.g. for the formation of human capital as in our model. In the next section we define equilibrium conditions and the balanced growth path.

2.4 Equilibrium conditions and the balanced growth path

An equilibrium allocation is defined as an allocation such that the firm maximizes profits implying that factor prices equal their marginal products (equations (7) and (8)), the household solves (1) subject to (2) and the budget constraint of the government (11) is fulfilled and it sticks to the rule defined in (13).

Using (11) and (14) we get I_E in equilibrium. Inserting the resulting value in (10), the growth rate of human capital is described by the following differential equation,

$$\frac{\dot{h}_c}{h_c} = (\epsilon/S)\frac{((1-u)h_c)^\gamma}{h_c}\left(\left(\frac{uh_c}{K}\right)^\alpha K(\tau-\phi-\alpha(1-u)/u) - B(\beta-\tau r)\right)^{1-\gamma}, \tag{17}$$

with r given by (8).

The economy-wide resource constraint is obtained by combining equations (2) and (14) as

$$\frac{\dot{K}}{K} = \frac{Y}{K}\left(1+\phi-\tau+\alpha\frac{1-u}{u}\right) - \frac{C}{K} - \frac{B}{K}(r\,\tau-\beta)\,. \tag{18}$$

Thus, the economy is completely described by equations (9), (14), (17) and (18) plus the limiting transversality condition of the household.

A balanced growth path (BGP) is defined as a path on which all endogenous variables grow at the same rate, i.e. $\dot{K}/K = \dot{B}/B = \dot{C}/C = \dot{h}_c/h_c$ holds, and the intertemporal budget constraint of the government must be fulfilled. Since the difference between the interest rate and the growth rate on the BGP is strictly positive and constant, implying $\lim_{t\to\infty}\int_0^t (r(\mu) - g_y(\mu))d\mu = \infty$, and the government sets the primary surplus according to (13) with $\beta > 0$, any path which satisfies $\dot{K}/K = \dot{B}/B = \dot{C}/C = \dot{h}_c/h_c$ is associated with a sustainable public debt.

To analyze our economy around a BGP we define the new variables $h \equiv h_c/K$, $b \equiv B/K$ and $c \equiv C/K$. Differentiating these variables with respect to time yields a three dimensional system of differential equations given by

$$\begin{aligned}
\dot{h} &= h(\epsilon/S)(1-u)^{\gamma}\left(u^{\alpha}h^{\alpha}(\tau-\phi-\alpha(1-u)/u) + (b/h)(\tau(1-\alpha)h^{\alpha}u^{\alpha}-\beta)\right)^{1-\gamma} + \\
&\quad h(c + b(\tau(1-\alpha)h^{\alpha}u^{\alpha}-\beta) - u^{\alpha}h^{\alpha}(1+\phi-\tau+\alpha(1-u)/u)), \qquad (19)\\
\dot{b} &= b(c + b(\tau(1-\alpha)h^{\alpha}u^{\alpha}-\beta) - u^{\alpha}h^{\alpha}(1+\phi-\tau+\alpha(1-u)/u)) - \phi u^{\alpha}h^{\alpha} + \\
&\quad (1-\alpha)u^{\alpha}h^{\alpha}b - b\beta, \qquad (20)\\
\dot{c} &= c\left((1-\tau)(1-\alpha)u^{\alpha}h^{\alpha} - \rho - u^{\alpha}h^{\alpha}(1+\phi-\tau+\alpha(1-u)/u) + c\right) + \\
&\quad c\,b(\tau(1-\alpha)h^{\alpha}u^{\alpha}-\beta). \qquad (21)
\end{aligned}$$

A solution of $\dot{h} = \dot{b} = \dot{c} = 0$ with respect to h, b, c gives a BGP for our model and the corresponding ratios $h^{\star}, b^{\star}, c^{\star}$ on the BGP.[2] In the next section we derive some economic implications of our model and numerically study its dynamics.

[2]The $\star$ denotes BGP values and we exclude the economically meaningless BGP $h^{\star} = b^{\star} = c^{\star} = 0$.

3 Implications of the model

3.1 Analytical results

To get insight into our model we first solve (21) with respect to c, insert that value in (20) and set equation (20) equal to zero giving

$$b^\star = \frac{\phi\, u^\alpha (h^\star)^\alpha}{\rho - \beta + \tau\, r}\,, \quad \text{with} \quad r = (1-\alpha)(h^\star)^\alpha\,. \tag{22}$$

From equation (22) we can derive a first result.

Since $b^\star$ is an endogenous variable, it can be positive or negative, with the latter implying that the government is a creditor. However, from an economic point of view a positive value of government debt is more realistic since most real world economies are characterized by public debt.

Assume that $\phi < 0$ holds. From an economic point of view, this implies that the primary surplus declines as GDP rises and the government raises its spending for education with higher GDP. In this case, β must be sufficiently large, more concretely $\beta > \rho + \tau\, r$ must hold, so that the debt-capital ratio is positive on the BGP, which can be seen from (22). This means that the reaction of the government to increases in public debt must be sufficiently strong such that a BGP with public debt is feasible. If this does not hold, i.e. if the reaction of the government to higher public debt is relatively small, the government must be a creditor for the economy to achieve sustained growth.

If $\phi > 0$, that is if the primary surplus rises as GDP increases, the contrary holds. In this case, β must not be too large, i.e. $\beta < \rho + \tau\, r$ must hold, so that sustained growth with positive public debt is feasible. This holds because a positive ϕ and a high β imply that the government does not invest sufficiently in the formation of human capital, which is the source of economic growth in our model. Consequently, if $\phi > 0$ and if β is relatively large, the government must be a creditor in order finance its investment in the schooling sector in order to achieve sustained growth. This can be seen from (17) which shows that for a positive ϕ and for $\beta > \rho + \tau\, r$, and thus $\beta > \tau\, r$, a negative government debt has a positive effect on the growth rate of human capital.

These considerations have shown that neither a too severe nor a too loose budgetary policy are compatible with sustained growth if the government is a debtor. On the one hand, if the government does not control public debt sufficiently, public debt becomes too high leading to a crowding-out of private investment, making sustained growth impossible. In this case, sustained growth is only possible if the government is a creditor.

On the other hand, the government must not conduct a too strict budgetary policy, implying that it does not invest enough in the formation of human capital, which is the source of economic growth. In this case, sustained growth is not feasible either and the government again must be a creditor, so that it can finance its investment in the schooling sector to build up human capital.

In the next section we resort to numerical examples in order to gain additional insight into our growth model.

3.2 Numerical analysis

3.2.1 Results for the model on the BGP

To analyze our model further, we resort to simulations. We do so because the analytical model turns out to become too complex to derive further results.

As a benchmark for our simulations we set the income tax rate to twenty percent, $\tau = 0.2$, and the elasticity of production with respect to physical capital is set to 30 percent, $1 - \alpha = 0.3$. The rate of time preference is set to 5 percent, $\rho = 0.05$. Further, 90 percent of the workforce is assumed to work in the final goods sector, $u = 0.9$, and 10 percent are in the schooling sector. The elasticity of human capital formation with respect to teachers is 75 percent, $\gamma = 0.75$ and we set $\epsilon / S = 0.15$.

In table 1 we report results of our simulations for different values of ϕ and for values of β which are smaller than the subjective discount rate ρ. g gives the balanced growth rate in percent and unstable means that two eigenvalues of the Jacobian matrix, evaluated at the rest point of (19)-(21), are positive.

Table 1

	$\beta = 0.015$				$\beta = 0.035$			
ϕ	$b^\star$	$h^\star$	g	Stability	$b^\star$	$h^\star$	g	Stability
$\phi = 0.05$	0.27	0.17	1.55%	unstable	0.43	0.17	1.35%	unstable
$\phi = 0.01$	0.05	0.18	1.72%	unstable	0.09	0.18	1.7%	unstable
$\phi = -0.01$	-0.05	0.18	1.79%	unstable	-0.09	0.18	1.81%	unstable
$\phi = -0.05$	-0.28	0.19	1.91%	unstable	-0.45	0.19	1.98%	unstable

Table 1 confirms the result derived for the analytical model that for small values of the reaction coefficient[3] β, ϕ must be positive so that sustained growth with positive public debt is feasible. In this case, the primary surplus must rise as GDP rises, otherwise endogenous growth with a positive government debt is not possible. If this does not hold, i.e. if ϕ is negative, the government must be a creditor and the level of public debt is negative. If ϕ is negative and if β is small, the government requires a too large fraction of the resources leading to a crowding-out of private investment, which can be seen from the economy wide resource constraint (18), so that sustained growth is not feasible in the long-run.

Further, one realizes that for a given value of β the growth rate rises as ϕ is decreased. From an economic point of view, a decrease in ϕ gives a deficit financed increase in public spending for eduction since it implies that the government reduces its primary surplus at the expense of other government spending, i.e. at the expense of spending for human capital formation in our model. Thus, higher spending for human capital enhances long-run growth even if financed through public deficits in this case. The reason for this

[3]Recall that in table 1 we have $\beta < \rho$ and, thus, $\beta < \rho + \tau r$.

outcome is to be seen in the fact that β, giving the reaction of the primary surplus to higher public debt, is small. A small β implies small negative feedback effects of higher debt and, consequently, a deficit financed increase in public spending for human capital formation raises the balanced growth rate. However, it must be pointed out that, if the government is a debtor, this policy is only feasible as long as the government raises its primary surplus with increases in GDP, i.e. as long as $\phi > 0$ holds. If ϕ is negative the government must be a creditor so that sustained growth is possible at all and so that this fiscal policy leads to higher long-run growth.

As to stability, the BGP is unstable in all cases. The eigenvalues are real with two being positive and one being negative. This means that there exists a one dimensional stable manifold. If one takes $h(0)$ and $b(0)$ as given, this implies that the set of initial conditions $\{h(0), b(0), c(0)\}$ lying on the stable manifold has Lebesgue measure zero. In this case, the economy can converge to the BGP in the long-run only if the government levies a lump-sum tax at $t = 0$ which is used to control $B(0)$ implying that $B(0)$, and thus $b(0)$, can be set. $B(0)$ and $C(0)$, then, must be chosen such that $b(0)$ and $c(0)$ lie on the stable manifold and these values are uniquely determined.

To gain further insight into our model we next set $\beta > \rho$. The results of the simulations are shown in table 2, where stable means that two eigenvalues of the Jacobian are negative or have negative real parts in case they are complex conjugate.

Table 2

	$\beta = 0.075$				$\beta = 0.1$			
ϕ	$b^\star$	$h^\star$	g	Stability	$b^\star$	$h^\star$	g	Stability
$\phi = 0.05$	-2.5	0.21	2.5%	stable	-0.45	0.19	1.98%	stable
$\phi = 0.01$	-0.38	0.19	1.95%	stable	-0.09	0.18	1.81%	stable
$\phi = -0.01$	0.31	0.17	1.51%	stable	0.08	0.18	1.7%	stable
$\phi = -0.05$	no BGP				0.39	0.17	1.41	stable

Table 2 demonstrates that for relatively large values of β the situation changes. In this case, the government must be a creditor if ϕ is positive. A large value of β implies that the government raises the primary surplus to a great extent as public debt rises.[4] Thus, a deficit financed increase in public spending for schooling, modelled by a decrease in ϕ, goes along with strong feedback effects of the higher public debt, so that in the end the economy ends up with a lower long-run growth rate. In this case, one can state that the government policy is too strict, in the sense that it pays too much attention to the control of public debt, instead of fostering economic growth by investing into education. Therefore, sustained growth with a positive government debt for large values of β is only possible if ϕ is negative, that is if the government does not raise the primary surplus as GDP rises but invests in education. If this does not hold, the government must be a creditor with a certain stock of wealth out of which it finances necessary investment in the schooling sector.

As concerns stability, the economy is stable for all situations considered in table 2. Two eigenvalues are negative or have negative real parts. Thus, the equilibrium is determinate since there exists a unique value for consumption at $t = 0$, which can be chosen freely, such that the economy converges to the BGP in the long-run. So, from table 1 and table 2 one can conclude that high values of β tend to stabilize the economy in the sense that the number of negative eigenvalues increases.

3.2.2 Sensitivity analysis of the dynamics

The dynamic system in the last subsection was characterized by a unique economic reasonable BGP with either a one- or two-dimensional stable manifold. In this subsection we want to analyze the dynamics of our growth model in more detail. It turns out that for $\beta < \rho$, existence of a BGP implies that it is unique.[5] For $\beta > \rho$ and for a sufficiently

[4]For $\beta = 0.1$, there exist two BGPs, however, one yields a negative $c^\star$ and does not make sense from an economic point of view.

[5]Technical details are in an appendix available on request.

large difference $\beta - \rho$, the BGP is also unique.

If the difference $\beta - \rho$ is small, we may observe a situation which is characterized by two BGPs, where one is unstable, except for a one-dimensional stable manifold, while the other is stable with a two-dimensional stable manifold. In this case, the economy converges to the second BGP unless the government intervenes at $t = 0$ by levying a lump-sum tax, which is used to control $B(0)$ such that the economy converges to the first BGP.

In addition, if we vary continuously the parameter β, the second BGP may undergo a Hopf bifurcation leading to persistent cycles.[6] In this case, the BGP is stable for certain values of β. If we decrease β the system becomes unstable and the real parts of the eigenvalues of Jacobian change sign from minus to plus. For a certain critical value of β, the eigenvalues are purely imaginary giving rise to a Hopf bifurcation and for slightly smaller values than the critical value, stable limit cycles can be observed.

In order to illustrate this phenomenon, we present some numerical examples. Setting $\phi = -0.01$ and $\rho = 0.05$ we get two BGPs for $\beta = 6.9\, e^{-2}$. The first yields a balanced growth rate of 2.9% and has one negative real eigenvalue. The second gives a balanced growth rate of 0.9% and has two complex conjugate eigenvalues with negative real parts. For $\beta_{crit} = 6.822717\, e^{-2}$ a Hopf bifurcation occurs at the second BGP leading to stable limit cycles,[7] which can be observed for values of β slightly smaller than β_{crit}.

Another example is obtained with $\phi = 0.05$ and $\rho = 0.01$. In this case, we get two BGPs for $\beta = 1.076\, e^{-2}$. The first gives a balanced growth rate of 1.3% and has one negative real eigenvalue. The second gives a balanced growth rate of 1.1% and has two complex conjugate eigenvalues with negative real parts. For $\beta_{crit} = 1.071655\, e^{-2}$ a Hopf bifurcation occurs at the second BGP leading to stable limit cycles, which can be observed for values of β slightly smaller than β_{crit}.

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[6] For a statement of the Hopf bifurcation theorem see e.g. Hassard et al. (1981).

[7] For those computations we used the software LOCBIF, see Khibnik et al. (1993).

For the parameter values in table 2 there exists only one reasonable BGP. However, we can also find a Hopf bifurcation, leading to stable limit cycles, with $\phi = -0.05$ for $\beta_{crit} = 8.481129\, e^{-2}$. For $\beta < \beta_{crit}$, the BGP first becomes unstable and, then, does not exist any longer as β is further decreased.

Before we give an economic interpretation of the limit cycles we should like to point out that, as in the last subsection, higher values of β tend to stabilize the economy, which seems to be intuitively clear. Thus, the stronger the reaction of the government to higher public debt, more concretely, the stronger the increase in the primary surplus as public debt rises, the more likely it is that the dynamic system describing the economy is stable. So, our examples in this subsection show that the economy is locally stable for relatively large values of β. As β is continuously decreased, the BGP looses stability and, before becoming unstable, produces limit cycles via a Hopf bifurcation.

In figure 1 we show the limit cycle for the second example[8] in the $(c-h-b)$ phase space, where the orientation is counter clockwise as indicated by the arrow.

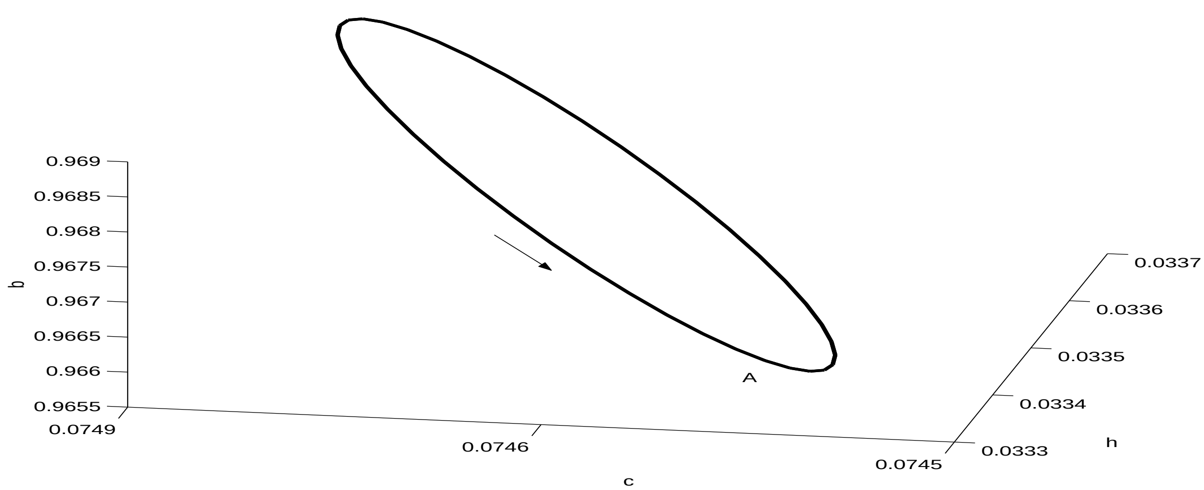

Figure 1: Limit cycle in the $(c-h-\ b)$ phase space.

[8]To detect the cycle we used the software CL_ MATCONT, see Dhooge et al. (2003).

To interpret the limit cycle we take point A in figure 1 as starting point. At that point h stops declining and begins to rise. An increase in h implies that the return to private capital rises leading to rising growth rates of consumption and of physical capital. However, since higher consumption goes at the expense of investment, the private capital stock does not rise as much as consumption, so that the ratio $c = C/K$, increases. It should also be noted that c lags behind h which is seen in figure 1, where h begins to rise while c still declines at point A.

The increase in the return to capital, caused by the rise of h, also leads to higher interest payments of the government. This generates higher public deficits which raise the growth rate of government debt. The latter rises stronger than private capital so that the ratio $b = B/K$ increases, too. With higher public debt the primary surplus rises, implying that the government can invest less resources in the formation of human capital so that, in the end, the ratio of human to physical capital h declines again. That is, as a result of higher public debt the feedback effect of public debt becomes effective, which leads to a decrease in government spending for schooling. When deficits have been sufficiently reduced, the government can spend more for schooling and the growth rate of human capital rises again leading to increases in h.

4 Conclusion

This paper has presented an endogenous growth model with human capital where the government finances educational spending which fosters the formation of human capital. The government may run deficits but it has to increase the primary surplus as public debt rises so that the path of public debt remains sustainable.

The analysis has demonstrated that a loose fiscal policy, where the government does not pay great attention to stabilizing debt, does not permit sustained growth in the long-run, unless the government is a creditor. In this case, there is a crowding-out of private investment and sustained growth is not feasible, unless the government is a creditor and lends to the private sector, so that the latter can finance necessary investment in physical capital. On the other hand, if the government puts a large weight on debt stabilization and does not invest sufficiently in the formation of human capital, sustained growth is not possible either, unless the government is again a creditor. In this case, the government must use its wealth in order to finance necessary investment in the formation of human capital.

As concerns stability, our analysis has shown that a strong rise in the primary surplus as a reaction to higher public debt stabilizes the economy. In addition, we could show that for certain values of the reaction coefficient, the economy displays persistent cycles of economic growth rates. If the reaction coefficient is set to a larger value, the economy stabilizes and converges to the constant balanced growth rate, if the reaction coefficient is set to a lower value, the economy becomes unstable.

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